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Global Financial Crisis and Multiscale Systematic Risk: Evidence from Selected European Stock Markets

A. Alexandridis

University of Kent, School of Mathematics, Statistics and Applied Science,
Cornwallis Building Canterbury, Kent, CT2 7NF UK

E-mail: A.Alexandridis@kent.ac.uk

Corresponding author

M. S. Hasan

University of Kent, Kent Business School
Canterbury, Kent, CT2 7PE, UK

E-mail: M.S.Hasan@kent.ac.uk

Abstract - In this paper, we have investigated the impact of the global financial crisis on the multi-horizon nature of systematic risk and market risk using daily data of eight major European equity markets over the period of 2005-2012. The method is based on a wavelet multiscale approach within the framework of a capital asset pricing model. The sample covers pre-crisis, crisis and post-crisis periods with varying experiences and regimes. First we investigate for possible contagion effects of the U.S. crisis to the European stock markets and then we perform a local analysis of each European stock market separately. Empirical results demonstrate that beta coefficients have a multiscale tendency in sample countries and betas tend to increase at higher scale (lower frequencies) for the whole period. However, the size of betas and R^2 s tend to increase during the crisis period compared to the pre-crisis period. The multiscale nature of the betas is consistent with the fact that stock market investors have different time horizons due to different trading strategies. Our results based on scale dependent value at risk (VaR) suggest that market risk tends to be more concentrated at lower time scale (higher frequencies) of the data. Moreover, the scale-by-scale estimates of VaR have increased almost three fold for every market during the crisis period compared to the pre-crisis period. Finally, we have presented an approach for accurately forecasting time-dependent betas and VaR using wavelet networks.

Key Words: global financial crisis, value at risk, multiscale systematic risk, CAPM, wavelet analysis, wavelet networks

JEL Classification: C22; G15

1. Introduction

There is tremendous interest among financial analysts, researchers, policy makers and the general public regarding the impact of the recent United States subprime crisis on the global financial markets ensued by a prolonged and deep global recession. The global financial crisis in 2008-2009 triggered by the subprime crisis led to a progressive

deterioration of the investment situation and financial climate around the globe, in general, and European economies in particular.¹

Although the major financial US institutions, such as New Century Financial, US holding of HSBC, and the world's top five investment banks suffered huge losses in the subprime mortgage and collateralized debt obligation (CDO) transactions by summer 2007, the world financial system observed a period of relative calm with some optimism regarding the outcome of the ongoing crisis until the eight months of 2008². Figure 1 presents a cursory example of several major banks' exposures to AIG during the time of financial crisis for readers to understand the magnitude and extent of the problem inherent in the systemic risks associated with the financial system and institutions.

The subprime mortgage crisis eventually erupted when first, major US financial firms, such as Lehman Brothers and AIG, and then European financial institutions, such as Northern Rock, Fortis, Dexia, and a number of Icelandic banks, showed signs of insolvency.³ The crisis exposed the inherent vulnerabilities, systemic risks and a catalogue of regulatory failures in the global financial services industries. The meltdown of the subprime crisis of 2007 exerted a meteor shower effect across the world's stock market by the fourth quarter of the 2008. In the last quarter of 2008, the stock markets of both developed and emerging economies experienced large decline in prices of securities.⁴ Figure 2 presents movements of market indices of stock in four

¹ For example, see the interim report title, "Assessing the impact of the current financial and economic crisis on global FDI flows", UNCTAD, January 2009.

² Dowd (2009) noted that the size of the collateralized debt obligations (CDO) market in 2007 was around \$500 billion, and then notional principal of the Credit Default Swaps (CDS) market by the end of 2007 was around \$60 trillion.

³ In a revised estimate of the International Monetary Fund, large US and European banks are expected to lose nearly 2.6 trillion from 2007 to 2010 where the US banks' forecasted loss tends to reach \$1 trillion and the European banks losses were expected to hit \$1.6 trillion (see Choudhry and Jayasekera, 2012).

⁴ Batram and Bodnar (2009) noted that the global equity market which stood at an all-time high of \$51 trillion in October 2007, dropped to \$22 trillion by the end of February 2009.

countries, namely, United Kingdom, Germany, Greece and Spain, during the period of 2005-2012 which uniformly demonstrates a sharp decline of share prices during the period of September-November, 2008 for all countries. Although the stock markets of United Kingdom and Germany exhibit an upward trend after November 2008, the stock markets of Greece and Spain show a persistent downward trend in their share prices. It is clearly evident that the Global financial crisis exerts an adverse impact on both systematic and market risks for all those countries.

One fruitful way to assess the impact of the global financial crisis, and contagion is to proceed with an investigation of stock markets' responses in terms of their effects both on systematic and market risks in highly correlated markets linked with trade and investment. Therefore, in this paper, we are investigating the impact of the global financial crisis on the stock markets of selected European markets, such as France, Germany, Greece, Italy, Netherlands, Portugal, Spain and the United Kingdom within the framework of Capital Asset Pricing Model (CAPM). The stock exchanges of these countries represent major exchanges within the European Union (EU) in terms of both market capitalisation and trading volume⁵. The behavior and performance of the CAPM during the pre-crisis, crisis, and two post-crisis periods provides a convenient and powerful framework for an empirical assessment of the impact of the crisis on the European stock markets.

Eichengreen *et. al.* (2012) investigated the impact of subprime crisis on the global banking system using dynamic factor model. The study employed principal components analysis to identify common factors in the movement of banks' credit default swap (CDS) spread. The study found that the share of the variance accounted

⁵ Furthermore, the US and European investors hold a large amount of financial assets in their portfolios by investing in ADR, GDR, country fund and direct participation in both markets. Given this closer relationship, investors and financial institutions from a number of European countries suffered huge losses in the US real estate market.

by common factors rose steadily to exceptional level from the outbreak of the subprime crisis which reflected the heightened funding and counterparty risks coupled with the deterioration of banks' loan portfolio. Vo (2014) utilized coexceedance approach to examine financial contagion in Euro Area and South Asian markets using a framework of multinomial logit regression model and daily data spanning the period January 2007 to March 2013. Exceedances are defined as extreme negative returns that are below a certain threshold (i.e., 5% bottom tail) in one country, whereas coexceedances refer to the joint occurrences of exceedances in two or more markets. The study documented evidence of coexceedances during global financial crisis and Eurozone crisis. Choudhry and Jayasekera (2013) reported that during the turbulent period of global financial crisis, betas increased for most firms in the UK from the pre-crisis to the crisis period and the level of market efficiency declined significantly from the pre-crisis to crisis period. Given the anecdotal evidence of significant deterioration of systematic risk and market risk, the findings from these studies indicate that the extent of co-movement in stock markets points to tendencies of the degree to which the global financial system is perceived to be tied to common factors. Consequently, CAPM and International Capital Pricing Model (ICAPM) provide an appropriate methodological framework to approximate the heightened systematic risk underlying the deterioration of common factor in such turbulent market condition.

In this paper, we have employed a recent and powerful method to estimate the systematic risk of CAPM using wavelet analysis (WA) to examine the meteor shower effects of the global financial crisis. One recent research strand of CAPM has built an empirical modeling strategy centering on the issue of the multiscale nature of the systematic risk using a framework of WA (Fernandez 2006, Gencay *et al.* 2005, Masih *et al.* 2010, and Norsworthy *et al.* 2000). The wavelet analysis provides a powerful tool

to decompose time-series data into orthogonal components with different frequencies and the method can accommodate structural change, discontinuity and regime shifts. The multi-horizon nature of the systematic risk or beta encompasses a range of ongoing research issues, such as time-varying beta, instability of beta, and varying behavior of beta with respect to return interval. Therefore, we have employed a wavelet approach to estimate and analyze the systematic risk and market risk on a scale-by-scale basis. The rationale and motivation for the use of a wavelet approach is discussed in the following sections. In our analysis we first investigate for possible contagion effects of the U.S. crisis to the European stock markets and then we perform a local analysis of each European stock market separately by applying a national CAPM. Our results suggest that the beta coefficients have a multi scale dependency and tend to increase at higher scale making CAPM predictions more meaningful for investment horizons of 8-16 days. In addition, the market risk tends to be concentrated at lower time scales. Finally, we have applied a new class of artificial neural networks, namely Wavelet Networks (WNs), in order to study if the dynamics and the multiscale nature of the systematic risks can be captured and forecasted. Our results indicate that WNs constitute an accurate tool for forecasting the multisacle nature of the systematic risk.

The paper is organised as follows. Section 2 provides a brief literature review surrounding the research strand of the stock market linkage and propagation, global financial crisis and the multiscale nature of systematic risks. Section 3 presents the model and furnishes a discussion on the methodology. Section 4 provides data description. Sections 5 and 6 discuss empirical results. In section 7 WNs are used to forecast the betas and the R^2 . More precisely in section 7.1 a quick introduction to WNs is presented while in section 7.2 our empirical results are presented and discussed. Finally, in section 8 we conclude.

2. Literature review

2.1 Theoretical Underpinning and Implication

The notion and significance of equity market linkage, crisis, contagion and spillovers spawned considerable research both at a theoretical and an empirical level that spans over almost three decades following financial market liberalisation, globalisation and the development of communication and information technology.⁶ One of the principal conduits for stock market propagation is that as world equity markets are becoming more integrated, so individual stock prices share common stochastic trend(s), alternatively known as cointegration. The long-run co-trending properties of stock indices across markets indicate that stock prices in these markets are underpinned by the same economic growth factors that determine earnings and dividends. The application of International Capital Asset Pricing Model (ICAPM) yields theoretical predictions which are in accordance with common trend(s). The latent factor model and the recent dynamic factor model hypothesise that stock prices are driven by three factors: a world factor; a regional factor; and a local factor representing idiosyncratic risk. A special case of cointegration is contagion where markets become excessively coupled. Bekeart *et. al.* (2005) defined contagion as correlation which exceeds the correlation that is produced by market fundamentals. They distinguish between two factors underlying a stock return: the US equity market return and the regional equity market return. Within this framework, the magnitude

⁶ Early research has explored stock market interdependence in terms of the dynamics of the conditional first moment (mean) of the distribution of returns. For example, see, Becker *et. al.* (1990), Choudhry *et. al.* (2007) and a list of references therein, and Masih and Masih (2002) and a list of references therein. A second research strand has also focused on the dynamics of stock market interdependence but in terms of 'both' conditional first moment and second moment of the distribution of returns, i.e., mean and volatility spillovers across markets. For example, see Engle *et. al.* (1990), Engle *et. al.* (2012) and a list of references therein, and Theodossiou and Lee (1993). A third research strand has been concerned with the issues of crisis and contagion. For example, see Classens and Forbes (2001), Forbes and Rigobon (2002), Dungey and Martin (2007), Dungey and Tambakis (2005), and Engle *et. al.* (2012) and a list of references therein.

and structure of correlations are examined, consequent upon a change in the volatility factor and factor sensitivities.

Engle *et al.* (1990) proposed two hypotheses as to how volatility might manifest itself across trading centers. The ‘heat wave’ hypothesis asserts that volatility has only location-specific autocorrelation, such that a volatile day in New York, for example, would be followed by another volatile day in New York. The ‘meteor shower’ hypothesis asserts that intraday volatility extends from one trading center to another, so that a volatile day in New York, for example, would be followed by a volatile day in London. Engle *et al.* describe the meteor showers in the context of complete access to world-wide news in a market which allows for continuous trading. In this model, terrestrial geography plays no role in determining the impact of news on the volatility of financial markets. In such a market, volatility spillovers occur when uninformed liquidity traders and investors with heterogeneous priors cannot efficiently absorb private information in the price formation of securities.

Given the theoretical underpinning and implications for efficiency, understanding the behavior of stock market propagation is important for several reasons. First, comprehending the behavior of stock market propagation is important to the investors and financial practitioners for valuing securities, executing hedging strategies and taking asset allocation decisions. Second, information on stock market propagation and volatility is needed by regulators of financial industries for the calculation of Minimum Capital Risk Requirement (MCRR), stress test and scenario analysis, based on value at risk and/or extreme value models. The information is of particular relevance for policy-makers with scope for intervention in financial markets and regulation of equity markets.

2.2 Empirical Studies on Global Financial Crisis

In the case of the recent global financial crisis, the problem is considered to have emanated from the toxic securitized markets, with consequent spillovers to the derivative markets, via, for example, Credit Default Swap (CDS), and to equity markets, by virtue of the meteor showers on the global financial markets. Consequently, a large body of empirical literature has accumulated in recent years regarding the causes and consequences of the global financial crisis on the global financial markets.⁷ The literature review which follows here constitutes merely a few representative sample of research that has been focused on the financial markets regarding the causes and consequences of the global financial crisis.

Dorn (2009) contended that U.S. housing policy, along with securitization and easy money contributed to the asset price bubble in the housing market.⁸ The role of government-sponsored enterprises, flawed financial-risk models, lax regulatory framework, inadequate credit rating and innovations that allowed banks to overleverage-all these factors in a body contributed to the sub-prime crisis. Schwartz (2009) argued that the process of asset securitization produced products that were difficult to price. Calomiris (2009) argued that inadequate or inappropriate regulation contributed to the subprime crisis by allowing banks to maintain insufficient amounts of equity capital per unit of risk undertaken in their subprime holdings.

Eichengreen *et. al.* (2012) investigated the impact of subprime crisis on the global banking system by examining the movement of banks' credit default swap spread. The study reported that share of the variance accounted by common factors was at 62% prior to the outbreak of the subprime crisis. During the period of July 2007 to September

⁷ Interested readers are referred to samples of few articles from special issues of the following journals on this topic: Applied Financial Economics, Vol. 20, Issue 1-2, 2010, Cato Journal, Vol 29, Issue 1, Winter 2009, Journal of International Money and Finance (in press).

⁸ Calomiris (2009) noted that total subprime and Alt-A originations grew from \$395 billion in 2003 to \$715 in 2004 and increased to \$1,005 billion in 2008.

2008, the share of common factor rose to 77%. Banti *et. al.* (2012) using proprietary data from a large investment bank reported that the magnitude of liquidity risk premium increased substantially after the collapse of Lehman Brothers during the period of financial crisis. Vo (2014) found evidence of coexceedances during global financial crisis and Eurozone crisis periods. The study reported that changes in exchange rates, the volatility of regional stock markets, the change in US long term interest rates, the TED spread and VIX are strongly significant to explain the probability of the joint occurrences of heavy losses in the Euro Area. Vardar and Aydogan (2014) examined the existence of dynamic linkages among the major emerging stock markets, namely Brazil, Hungary, China, Taiwan, Poland and Turkey, as well as developed markets, such as the US, the UK and Germany during the period 2004-2013 by splitting the sample in two sub-periods: before crisis and after crisis. Their empirical findings indicate that the direction of the long-run relationship varies across sub-periods; during the crisis and post crisis periods, the stock market interdependence increased. They also reported evidence of herding behavior of investors during the period of stock market crash.⁹

In recent studies Choudhry and Jayasekera (2012, 2013) investigated the anomalous behavior of stock prices and asymmetric response of time-varying beta using the data from US-UK bank stocks and UK stock markets during the period of global financial crisis, respectively. Their empirical results reported that the level of market efficiency declined and the time-varying betas for individual firms increased significantly during the crisis period. They rationalized the anomalous behavior of stock prices in terms of two competing hypotheses, i.e., market efficiency hypothesis

⁹ Given the evidence of increasing dynamic co-movements of stock markets during the periods of crisis and post crisis, Vardar and Aydogan (2014) suggested that opportunities of the international risk diversification and achievement of greater portfolio returns had been reduced for the international investors.

and behavioral finance based explanation. The market efficiency hypothesis (EMH) predicts that beta of individual stock rises (fall) in response to abnormally negative (positive) returns as an asymmetric response to good and bad news. Regarding the behavioral finance explanation, there exists plethora of literature which presents evidence of over/under reaction of stock prices to new information.¹⁰ In a separate study Choudhry and Jayasekara (2014) investigated the asymmetric effect of news on the time-varying beta on selected thirteen banks from seven European countries using the daily data spanning the period 2002 to 2013. Their empirical evidence again documented evidence of declining market efficiency, increased size of time-varying beta and asymmetric effect in time-varying beta during the period of financial crisis. Although their results demonstrate some evidence of market efficiency through non-market shock, European banks have shown evidence of significant amount of uncertainty leading to asset mispricing through market shocks.

There is a rich array of literature on the asymmetric effect of good and bad news on stock prices. For example, see Black (1976), Christie 1982), Cho and Engle (1999), and Glosten *et. al.* (1993); see Choudhry and Jayasekera (2012, 2013, 2014) for a list of reference. The explanation of asymmetric effect on time-varying beta emanates from two plausible sources, such as leverage based explanation and volatility based explanation. The leverage effect is due to the reduction in the equity value, which would raise the debt-equity ratio, hence raising the riskiness of the firm as a result of an increase in future volatility.¹¹ The volatility based explanation posits a positive relation between volatility and expected risk premium.¹² An increase in volatility raises

¹⁰ For example, see Loughran and Ritter (1995), Dharan and Ikenberry (1995), Frazzini (2006); and for a list of reference see Choudhry and Jaysekera (2014).

¹¹ For example, see Black (1976), Cho and Engle (1999), Christie (1982), and Glosten *et al.* (1993).

¹² For example, see Poterba and Summers (1986), Bollerslev *et. al.* (1988), and Choudhry and Jayasekera (2013) and a list of reference therein.

the expected return by lowering the stock prices which in turn contributes to the asymmetric effect in volatility. Consequently, this effect in volatility is impacted upon the beta through an asymmetric effect. In a recent study Iqbal and Kume (2014) investigated the impact of the recent financial crisis on the capital structure decision of UK, French and German firms. Their results indicate that overall leverage ratios increased from pre-crisis (2006 and 2007) to crisis (2008 and 2009) period and then decreased in the post-crisis (2010 and 2011) period.

2.3 Empirical Studies on CAPM

Since the seminal contribution made by Sharpe (1964) and Lintner (1965), the notion and significance of the CAPM has spawned considerable research at both theoretical and empirical levels that spans almost six decades. According to CAPM, in a perfect capital market, the excess return of a stock or a portfolio of stocks (return over the riskless rate of return) should move in proportion to the market premium (market return over the riskless rate of return). The proportionality factor known as ‘beta’ (β) captures the ‘systematic risk’ of the market. Although early research during the 1970s was supportive of the theoretical prediction of the CAPM, later studies during the 1980s and 1990s yielded mixed results. Empirical research aimed at testing the validity of the CAPM progressed and expanded through several distinct strands. Gençay *et al.* (2005) succinctly summarized those issues as: the stability of beta over time, borrowing constraints, the impact of structural change and regime switches, the effect of world markets and volatility, non-synchronous data issues, time horizons of investors and the impact of return interval.

Previous studies suggest that the empirical validity of CAPM appears to depend on the return interval chosen albeit with mixed results. For example, studies of Kothari *et al.* (1995), and Handa *et al.* (1993) show that β s from annual returns produce stronger

relation between beta and average return than β s from monthly return. Frankfurter *et al.* (1994) contend that the mean and variance of β increases from daily returns to yearly returns. A study by Brailsford and Faff (1997) suggests that CAPM is rejected when daily returns data is used, while CAPM is accepted when weekly returns data is used. In contrast, Fama and French (1996) show that annual and monthly β s produce the same inference about the β premium.¹³

Given the mixed results regarding the inference about the CAPM and β s, and the multiscale nature of the systematic risk, in this paper, we have employed a powerful method to estimate the systematic risk of CAPM using WA to examine the meteor shower effects of the global financial crisis on selected European stock markets. WA provides an appropriate platform to investigate the multi-scale behavior of beta at different time horizons in a frequency domain framework. More precisely, we first investigate for contagion effects from the U.S. crisis to the European stock markets and then we apply locally the multiscale CAPM framework using WA.

Our analysis is also motivated by Fernandez (2006), Gencay *et al.* (2005), Masih *et al.* (2010), and Norsworthy *et al.* (2000), among others, who advocate the incorporation of different time scales using a framework of WA in the empirical reassessment of CAPM. As Masih *et al.* (2010) contended, the security market consists of thousands of traders and investors with different time horizons and strategies in their mind regarding the investment decision.¹⁴ Owing to different decision-making time

¹³ Several explanations are offered for the interval bias of systematic risk, such as infrequent trading, delays in information processing, increase of standard error of the beta as the return interval is lengthened, disproportionate move of covariance relative to the variance estimate in the measurement of beta, and seasonality. Masih *et al.* (2010) furnished a good discussion on the issue.

¹⁴ For example, within the speculator group, there are scalpers, day traders and position trader who act in the markets ranging from minute by minute, hour by hour, day by day, even month by month. Even within the three different types of participants, i.e., hedgers, speculators and arbitrageurs, in the derivative markets, there are long-horizon traders who concentrate on long-run price fundamental and there are short-term traders who respond to information within a short-term horizon [see, Conner and Rossiter (2005), and Fernandez (2008)].

horizons and strategies, among investors, the true dynamics of the relationship between stock returns and risk factors is likely to vary depending on the time horizon of the investors. In addition, even if investors agree on a well-diversified portfolio to be the proxy of market portfolio, their perception and measurement of the portfolio risk will not be the same. In this circumstance, financial analysts need to examine the behavior of systematic risk using a framework of different time scales or horizons in decision making process. Furthermore, Fernandez (2006) recommends the use of wavelet method as a suitable alternative to GARCH and GARCH-in-mean models to study the time-varying beta and time-varying risk premium. Wavelet analysis provides a robust approach under the conditions of structural break, discontinuity, non-normality and time-varying volatility.

3. Methodology

3.1 The Capital Asset Pricing Model

The choice of optimal portfolio in investment decision emanates from the consumption-saving-investment decision of a representative investor. The choice of the optimal portfolio is a function of both the risk-return possibility curve that is available in the market and the investor's utility function. The optimum consumption-saving-investment decision is obtained by setting the investor's subjective marginal rate of substitution (MRS) between risk and return equal to the slope of the risk-return possibility curve.

Both life-cycle and permanent income hypotheses utilize an inter-temporal optimization framework where our infinitely-lived representative household is faced with the following problem of maximization¹⁵:

¹⁵ This part draws extensively from Gencay *et al.* (2005) and Gausden and Whitfield (2000).

$$E_t \sum_{s=t}^{\infty} (1+\delta)^{-(s-t)} u(c_s) \quad (1)$$

subject to

$$E_t w_s = E_t [(1+r)w_{t-1} + y_s - c_s], \quad s = t, t+1, \dots, \infty \quad (2)$$

where E_t denotes mathematical expectations, conditional on all information available at t ; δ signifies rate of subjective time preference; $u(\cdot)$ implies single-period utility function; variables c , w , r and y denote constant-price consumer expenditure, real value of non-human wealth, real rate of interest, and real labor income, respectively.

Solving the first-order condition for a constrained optimization problem from the corresponding lagrangean function, and after certain manipulation, we may derive the following stochastic Euler equation (see Gausden and Whitfield, 2000):

$$E_t u'(c_{t+1}(1+\delta)/(1+r)) = E_t u'(c_t) \quad (3)$$

Equation (3) asserts that optimal consumption decision requires marginal utilities of adjacent periods to be proportional to one another. Assuming that consumer can allocate his wealth among $n-1$ risky assets with a $r_{i,t}$ rate of return and a riskless asset with a rate of return $r_{f,t}$, the resulting first order condition may be rewritten as:

$$E[u'(c_{t+1})]E[r_{i,t} - r_{f,t}] + Cov[u'(c_{t+1}), r_{i,t}] = 0 \quad (4)$$

The return from asset i must satisfy the following equation in equilibrium:

$$E[r_{i,t}] = r_{f,t} - \frac{Cov[u'(c_{t+1}), r_{i,t}]}{E[u'(c_{t+1})]} \quad (5)$$

If we further assume that the return of a benchmark market portfolio (provided by market index) is inversely related with the marginal utility of consumption in the next period, so that:

$$u'(c_{t+1}) = -\gamma r_{m,t} \quad (6)$$

for some positive γ .

It follows that $Cov[u'(c_{t+1}), r_{i,t}] = \gamma Cov[r_{i,t}, r_{m,t}]$ and allows us to rewrite equation (5) after certain manipulation as¹⁶:

$$E[r_{i,t}] = r_{f,t} + \frac{Cov[r_{i,t}, r_{m,t}]}{\sigma_m^2} [E[r_{m,t}] - r_{f,t}] \quad (7)$$

Equation (7) in estimable form yields the widely presented testing equation for the CAPM (Sharpe 1964, Linter 1965):

$$r_{i,t} - r_{f,t} = \beta(r_{m,t} - r_{f,t}) + \varepsilon_{i,t} \quad (8)$$

The CAPM predicts that the return to individual stock is a direct and linear function of the investments' systematic risk. Equation (8) is estimated at different time-scales. The beta is defined as:

$$\beta_t = \frac{Cov(r_{i,t}, r_{m,t})}{Var(r_{m,t})} \quad (9)$$

Where $Cov(r_{i,t}, r_{m,t})$ signifies the covariance between the return on asset i and the return on the market portfolio and $Var(r_{m,t})$ denotes the variance of the portfolio return. When beta is found to be more than unity, this suggests that the firm under study is perceived more risky than the market. Alternatively, if beta is greater than 1, the security is termed to be aggressive, and if it is less than 1, it is said to be defensive.

3.2 Wavelet Analysis

WA is an extension of Fourier analysis. The fundamental idea behind wavelets is to analyze according to scale. Low scale represents high frequency while high scales represent low frequency. The wavelet transform (WT) not only is localized in both time and frequency but also overcomes the fixed time-frequency partitioning. This means that the WT has good frequency resolution for low-frequency events and good

¹⁶ For a detailed derivation, see Gencay *et al.* (2005).

time resolution for high-frequency events. Hence, the WT can be used to analyze time series that contain no stationary dynamics at many different frequencies. Wavelet techniques are being used in finance, for detecting the properties of quick variation of values and it is a powerful tool for representing nonlinearities (Alexandridis and Zapranis 2014).¹⁷

The daily return time-series are represented by local information such as frequency, duration, intensity and time-position and by global information such as the mean states over different time periods. Both global and local information is needed for a correct analysis of the daily return time-series. Wavelets have the ability to decompose a signal or a time-series in different levels bringing out the structure of the underlying signal as well as trends, periodicities, singularities or jumps that cannot be observed originally.

As illustrated in Donoho and Johnstone (1994) the wavelet approach is very flexible in handling very irregular data series. Ramsey (1999) contends that WA has the ability to represent highly complex structures without knowing the underlying functional form, which is of great benefit in economic and financial research.

3.3 Maximal Overlap Discrete Wavelet Transform

Two versions of the WT can be distinguished. The continuous wavelet transforms (CWT) and the discrete wavelet transforms (DWT). The CWT can operate at every scale. However, an upper bound is determined since CWT is extremely computationally expensive. In order to reduce the computational burden, alternatively wavelet coefficients are calculated only on a subset of scale under the DWT method. In this study we use the Maximal Overlap DWT (MODWT). The MODWT is an extension of the classical DWT that has many desirable properties (Gencay *et al.* 2002, Percival

¹⁷ Fernandez (2006), Gencay *et al.* (2002, 2005), In and Kim (2006, 2007), Kim and In (2005, 2007), Maharaj *et al.* (2011), Masih *et al.* (2010), Norsworthy *et al.* (2000), Ramsey (1999), Rua and Nunes (2012) and Zapranis and Alexandridis (2008) are sample of recent few papers which have used wavelet approach to analyse financial time series.

and Walden 2000, and In and Kim 2007)¹⁸. First, the MODWT can handle any sample size of the data. Second, the MODWT does not suffer from sensitivity to the choice of a starting point for a time series. More precisely, in MODWT both wavelet and scaling coefficients are invariant to circularly shifting the original time series. Third, the detail and smooth coefficients of a MODWT multi-resolution analysis (MRA) are associated with zero phase filters. Hence, it is possible to align features in the MRA with the original time-series. Finally, the wavelet variance estimator is asymptotically more efficient than the same estimator based on the DWT. However, on the other hand the MODWT is more computationally expensive than the classical DWT¹⁹.

A time-series $f(t)$ can be written as a linear combination of wavelet functions as follows:

$$f(t) \approx \sum_k s_{J,k} \varphi_{J,k}(t) + \sum_k d_{J,k} \psi_{J,k}(t) + \sum_k d_{J-1,k} \psi_{J-1,k}(t) + \dots + \sum_k d_{1,k} \psi_{1,k}(t) \quad (10)$$

where J is the number of scales and k indicates the k^{th} coefficient. Following the notations from Fernandez (2006) the wavelet transformed coefficients $s_{J,k}, d_{J,k}, \dots, d_{1,k}$ can be approximated by the following integrals: $s_{J,k} \approx \int \varphi_{J,k}(t) f(t) dt$ and $d_{j,k} \approx \int \psi_{j,k}(t) f(t) dt$ where $j = 1, 2, \dots, J$. The functions $\varphi_{j,k}$ and $\psi_{j,k}$ are the

¹⁸So far, the MODWT was successfully applied in many studies in finance. For example, the MODWT was applied for the estimation of the hedge ratio in In and Kim (2006), while it was used in the estimation of the International CAPM in In and Kim (2007). The estimation of the systematic risk was studied in Gençay *et al.* (2002, 2005), Masih *et al.* (2010) and Rabeh and Mohamed (2011). In Maharaj *et al.* (2011), a comparison is made of developed and emerging equity market return volatility at different time scales. In Kim and In (2007), the relationship between changes in stock prices and bond yields in the G7 countries was studied. Finally, in Kim and In (2005) the relationship between stock returns and inflation is examined using the MODWT.

¹⁹In this study the LA8 (Least Asymmetric of length 8) wavelet transform filter is used. Our analysis is performed in 5 levels of the decomposition and the reflection method was used for the boundary conditions.

approximating wavelet functions. By setting $S_J(t) = \sum_k s_{J,k}(t) \varphi_{J,k}(t)$ and

$D_J(t) = \sum_k d_{J,k}(t) \psi_{J,k}(t)$ the original time-series can be reconstructed:

$$f(t) \approx S_J(t) + D_J(t) + D_{J-1}(t) + \dots + D_1(t) \quad (17)$$

This reconstruction is known as MRA. MRA is applied in order to reconstruct the original time-series from the wavelet and scaling coefficients. The elements of S_J are related to the scaling coefficients at the maximal scale and therefore represent the smooth components of $f(t)$. The elements of D_j are the detail (or rough) coefficients of $f(t)$ at scale j .

3.4 Computation of Wavelet Variance and Covariance

In order to estimate the wavelet-variance, the variance must be split into various parts, each one representing the variance at each scale. This wavelet-variance analysis shows us which scales are contributing significantly to the overall variability of the time-series (Percival and Walden, 2000). For a stationary process X , the variance σ_X^2 is given by:

$$\sigma_X^2 = \sum_{j=1}^{\infty} \nu_x^2(\tau_j) \quad (18)$$

where $\nu_x^2(\tau_j)$ is the wavelet variance for scale τ_j . Equation (18) is analogous to the relationship between the variance of a stationary process and its spectral density function (see for example, Fernandez, 2006; and Masih *et al.* 2010). An unbiased estimator of the wavelet variance is given by (Gencay *et al.* 2002):

$$\hat{\nu}_X^2(\tau_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^2 \quad (20)$$

where $\tilde{d}_{j,t}^2$ is the MODWT wavelet coefficients at scale τ_j , n is the sample size, L_j is the length of the scale τ_j wavelet filter and \tilde{N}_j is the number of the MODWT coefficients unaffected by the boundary.

Similarly, an unbiased estimator of the wavelet-covariance between two time-series X and Y is given by:

$$\hat{\nu}_{XY}^2(\tau_j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^{(X)} \tilde{d}_{j,t}^{(Y)} \quad (21)$$

Since the wavelet variance and wavelet covariance are known, under the CAPM the wavelet beta estimator for asset i at scale j is defined as:

$$\hat{\beta}_i(\tau_j) = \frac{\hat{\nu}_{R_i R_m}^2(\tau_j)}{\hat{\nu}_{R_m}^2(\tau_j)} \quad (22)$$

where $\hat{\nu}_{R_i R_m}^2(\tau_j)$ is the wavelet covariance of asset i and the market portfolio at scale j , and $\hat{\nu}_{R_m}^2(\tau_j)$ is the wavelet variance of the market portfolio at scale j .

Furthermore, the wavelet R^2 estimator for asset i at scale j is given by:

$$R_i^2(\tau_j) = \hat{\beta}_i^2(\tau_j) \frac{\hat{\nu}_{R_m}^2(\tau_j)}{\hat{\nu}_{R_i}^2(\tau_j)} \quad (23)$$

4. Data description

We are investigating the impact of the crisis on the stock markets of eight European markets. The selected markets are distinguished in two groups. The first group consists of four countries that at the moment face much European uncertainty and are under a rescue program and under the supervision of the International Monetary Fund (IMF) and/or the European Central Bank (ECB). These countries are: Portugal, Italy, Greece and Spain. On the other hand, the second group consists of four countries whose economies are traditionally considered strong and stable. These countries are:

Germany, Netherlands, UK and France. The selected countries represent major exchanges within the EU in terms of both market capitalization and trading volume.²⁰

Our data set includes the daily values of the main stock index in each country from June 1, 2005 to September 10, 2012 as well as the daily stock prices of the stocks that constitute each index²¹.

In this study we estimate the beta of a risky asset at different time-frequencies and in different time-periods in order to obtain an estimate of the impact of the U.S. crisis in the systematic risk in these markets.

Our data set is split into four different periods. The first data set corresponds to the pre-crisis period and includes daily stock values from June 1, 2005 to July 31, 2007. The second data set represents the crisis period and it is the dataset ranges from August 1, 2007 to September 30, 2009²². The third data set represents the post-crisis period in U.S. and the beginning of the crisis in Europe, October 1, 2009 to November 30, 2011. Finally, there is a fourth data set from December 1, 2011 to September 10, 2012 that represents the current situation in Europe.

In order to avoid survivorship bias only the stocks that survive for each sample period are examined.²³ Daily return series for each stock as well as the market index were collected from each stock market. This resulted in 564 values for the first sample,

²⁰ The value of stock market capitalization for markets of the United Kingdom, France, Germany, Netherlands, Spain, Italy, Portugal and Greece in 2012 are 3019, 1823, 1486, 651, 995, 480, 66 and 45 billion US dollars, respectively.

²¹ The eight indices are the following: AEX25 from Netherlands, FTSE/ATHEX 20 from Greece, CAC 40 from France, DAX 30 from Germany, FTSE 100 from UK, IBEX 35 from Spain, MIB 40 from Italy and PSI 20 from Portugal.

²² The mortgage financial crisis usually starts from the August 1, 2007 and continues until July 31, 2009.

²³ Pre-crisis: Netherlands: 23, Greece: 19, France: 37, Germany: 30, UK: 87, Spain: 26, Italy: 32, Portugal: 15.

In-crisis: Netherlands: 23, Greece: 20, France: 40, Germany: 30, UK: 94, Spain: 30, Italy: 36, Portugal: 17.

First post-crisis: Netherlands: 23, Greece: 20, France: 40, Germany: 30, UK: 97, Spain: 31, Italy: 39, Portugal: 18.

Second post-crisis: Netherlands: 25, Greece: 20, France: 40, Germany: 30, UK: 100, Spain: 35, Italy: 40, Portugal: 19.

566 for the second, 565 for the third, 203 for the fourth, giving a total of 1898 values. The logarithmic returns of the stocks, $r_{i,t}$, and of the market index, $r_{m,t}$, were computed.

For the estimation of model (8) the risk-free rate of return is proxied by the daily rate of return from 1-month Euribor offer rate for all countries with an exception in the case of the U.K. where it is represented by the daily rate of return from the 1-month UK Treasury bill rate.

5. Empirical results

In this section the contagion effects of the U.S. crisis on eight European stock markets will be studied. Then, our analysis will be focused locally in each country by studying the multiscale systematic risk.

In Table 1 the correlation between the S&P 500 and the selected European stock markets under study is presented. The correlation was estimated in different time-periods. During the pre-crisis period the correlation coefficients between the U.S. and European markets range around 0.96 for all countries. Focusing on the in-crisis time period, we observe a slight increase in the correlation, as it is expected. In the first post-crisis period where the U.S. economy started to recover while in the Eurozone the first stages of the crisis, it is evident that the correlation significantly decreased. The correlation between the market index of the weak economies and the S&P 500 is negative (Greece, Spain, Italy and Portugal), while it is 0.83 and 0.87 for Germany and UK respectively. Finally, focusing on the second U.S. post-crisis period when the crisis in Europe was deeper, we observe an increase in the correlation with an exception of Spain and Portugal where the correlation remains negative.

As it can be seen, correlation coefficients show significant instability over the sub-periods. The same correlation rises during the crisis period, and then decreases with change in sign and magnitude for four countries during the first post-crisis period.

Therefore, it is evident that the U.S. crisis had an effect on the performance of the major European stock markets. Next, we will focus on the local CAPM of each country.

In Table 2 to Table 5 the beta and R^2 at each scale j are presented while in the last two columns the beta and R^2 from the raw data are presented. In each scale the average beta and the average R^2 of all stocks of each index are presented. The raw values refer to the classic CAPM estimation using regression. The raw values of beta and R^2 were estimated for comparison reasons. In all tables the raw values are very close to the average beta and R^2 across all scales. Our analysis was performed in a depth of 5 scales. Scale 1 corresponds to periods of 2-4 days, scale 2 to 4-8 days, scale 3 to 8-16 days, scale 4 to 16-32 days and scale 5 to 32-64 days.

In Table 2 the results for the pre-crisis period are presented. It is clear that the linear relationship between an individual stock and the market portfolio becomes stronger as the scale increases. However, in most cases a slight decrease is observed at scale 5. In other words the maximum values of beta and R^2 are observed in scales 3 and 4. Our results accord well with Gencay *et al.* (2005) and Fernandez (2006). The results for all countries are similar. The mean betas in each scale are close to 1 and increase in higher scales. We also observe that the systematic risk of almost all stocks and proposed portfolio is less than one in the markets of Greece, Netherlands, Germany, Italy and Portugal for 2-64 days horizon. This suggests that the benchmark market indices have a reduced impact on assets in these markets in the short- to intermediate-run horizons. Similarly, the R^2 increases as the scale increases. The R^2 ranges from 0.12 at scale 2 for Portugal to 0.50 at scale 5 for Spain. The lower values of R^2 are observed in Portugal, Netherlands and Greece while the highest values are observed in Spain, Germany and France. In addition, we also observe that countries with greatest

sensitivities to the market portfolio in the raw data are Spain and UK, followed by France.

Our analysis in Table 3 reflects the results during the in-crisis period. Closer inspection of Table 3 reveals that betas have increased for the stock markets of France, Netherlands and Portugal. The increased magnitudes of betas reflect heightened sensitivity of financial market to the whole range of economic and financial variables, and incomplete knowledge regarding the magnitudes to toxic asset positions in the early stage of the crisis. The evidence of increased betas during the financial crisis era is consistent with the findings of Choudhry and Jayasekara (2012, 2013, 2014) and others.²⁴ During the crisis, negative information led to a freeze in several markets which may have led to decrease in magnitudes of betas in other markets, i.e., the markets for Greece, Italy, Spain and the UK. However, the R^2 is increased for every country. The lower values of R^2 observed in UK, Germany, Italy, and Portugal are 0.35, 0.35, 0.37 and 0.38, respectively while the R^2 for the remaining countries is over 0.40 and up to 0.53 for Spain. In addition, in contrast to the remaining countries, the beta for Greece fluctuates between 0.83 and 0.84 for the first four scales and then increases to 0.92 in the last scale. For the remaining countries the maximum beta is observed at scales 3 and 4 while the minimum, usually, at scale 1.

Next, we focus on Table 4 where the results during the first post-crisis period are presented. This period reflects the end of the US crisis and the beginning of the European crisis. Our results indicate that the betas in almost every country are almost 1 for each scale although a slight increase is observed at higher scales. The increased sizes of the betas again indicate the heightened sensitivities induced by the uncertainty

²⁴ Rua and Nunes (2012) noted that beta tends to rise during crises period, such as the Mexican crisis in 1994, Emerging market crisis in 1998, Turkish crisis in 2006 and recent global financial crisis.

of the Eurozone crisis in these markets. The R^2 has increased in each country and it is 0.51, 0.48, 0.61, 0.51, 0.44, 0.57, 0.53, 0.47 in Netherlands, Greece, France, Germany, UK, Spain, Italy and Portugal, respectively for the raw data. Again the maximum betas are observed at scales 3 and 4 while the minimum at scale 1 for most countries. For all countries the R^2 increases from scale 1 to scale 3 and then starts to decrease until scale 5.

Table 5 presents the results of our analysis in the last time-period which reflects the current situation in Europe. The results are similar to the ones presented in Table 4. However, an increase is observed in the beta values of Netherlands, Greece, France, and Italy. On the other hand, the betas in Germany and UK remained almost the same. On the contrary, the R^2 is reduced for every country. The maximum betas are observed at scales 3 for France, UK and Spain; at scale 4 for Greece, Portugal and Netherlands; at scale 5 for France, Italy and Germany. For all countries the R^2 increases as we move from lower scales to mid-scales and then it decreases at higher scales except for Greece. Overall, values of betas in Eurozone countries, such as France, Greece, Italy, Netherlands, and Portugal, have increased during the period of Eurozone crisis.

In summary, the values of betas have increased at low frequencies (higher scale) across periods and markets. Values of betas have changed during global financial crisis period relatively to the pre-crisis period. During the period of Eurozone debt crisis, values of betas have increased for the majority of the sample Eurozone countries. The increase of multi-scale betas during periods of global financial crisis and Eurozone crisis may be induced by a combination of leverage effect and asymmetric response of the market to bad news. In addition, a rich array of literature in behavioral finance presented evidence of under/over reaction of stock prices to information in such turbulent market condition. The evidence of asymmetric effect in time-varying beta is

consistent with the findings of Choudhry and Jayasekera (2012, 2013, 2014) and the evidence of leverage effect accords well with the finding of Iqbal and Kume (2014) in their study of markets from the UK, France and Germany. Furthermore, as Braun, Nelson and Sunier (1995), and Ball and Kothari (1989) contended that an increase (decrease) in market shocks to the firm increase (decrease) the beta and lead to a rise (fall) in expected return in market. Therefore, asymmetry in volatility in these markets during the crises periods led to asymmetry in time-varying beta.

Chiang *et. al* (2007) and Syllingnakis and Kouretas (2011) provide the evidence of financial contagion due to herding behavior during the financial crisis.²⁵ In normal market conditions, investors and traders use on some occasions technical analysis such as momentum trading to generate above average market return. However, during the crisis, negative information led to a freeze in several markets or may have led to a disposition effect. As mean betas in each scale are around 1 or slightly above 1 and increase in higher scales, investors and traders should employ a contrarian trading strategy across scales. The long-term investors may utilize a buy and hold strategy in such a bearish market. Once the market rebound to their long-run mean values, the investors may resort to momentum trading strategy.

Due to space limitations, the results presented in Table 2 to Table 5 are the estimated averages for each country. As a result the values of betas are close to 1 as it was expected. In the Appendix the analytical results are presented for the PSI-20 index in order to provide a better understanding of the changes of the betas according to scale. More precisely the betas and R^2 for each stock in the PSI-20 index are presented together with the raw estimates of betas and R^2 . The results from the remaining

²⁵ Using the example of US stock market crash of October 1987, Lin *et al.* (1994) argued that price movements driven by fads and herd instinct have the capability of being transmitted across borders when speculative trading and noise trading occur in international financial markets.

countries and indices are available from the authors upon request. For example, Appendix Table 12 indicates that a firm with abbreviated listing BCP (Banco Comercial Portugues) shows greatest sensitivity to the market portfolio in Portugal, followed by PTC (Portugal Telecom Sgps). The firms with the abbreviated listing ALT (Altri Sgps) and BES (Banco Espirito Santo) demonstrate that long-term traders are most risk averse, at scale 5. Overall, in firms with low betas across scales traders are ‘amateur’ of risk.

6. Value-at-Risk at different time-scales.

In this section, we focus on the estimation of the Value-at-Risk (VaR). VaR is a very popular measure that describes the market risk. VaR measures the amount that an investor can lose with a given probability over a certain time horizon.

We construct a portfolio where individual company stocks within each country constitute the portfolio. For simplicity we assume an equally weighted portfolio of k assets where ω is vector that contains the portfolio weights, i.e. a $k \times 1$ vector which each element is $1/k$. Then, the ratio:

$$\frac{\sigma_m^2(\tau_j) \left(\sum_{i=1}^k \beta_i(\tau_j) / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^k \sigma_{\varepsilon_i}^2(\tau_j)}{\sigma_m^2 \left(\sum_{i=1}^k \beta_i / k \right)^2 + \frac{1}{k^2} \sum_{i=1}^k \sigma_{\varepsilon_i}^2} \quad (26)$$

is an estimate of the contribution of scale j to total VaR of an equally weighted portfolio (see for example, Fernandez, 2006; and Masih *et al.* 2010), where

$$\sigma_{\varepsilon}^2(\tau_j) = \sigma_i^2(\tau_j) - \beta_i^2(\tau_j) \sigma_m^2(\tau_j) \quad (27)$$

and $\sigma_i^2(\tau_j)$ is the variance of stock i at scale j , $\beta_i(\tau_j)$ is the beta of stock i return at scale j and the variance of the market portfolio at scale j is given by $\sigma_m^2(\tau_j)$.

In Table 6-Table 9 the $VaR(a)$ at different time scales for an equally weighted portfolio is presented for the four different time periods. The initial value of the portfolio is 1 unit of the specific market's currency invested in 1-day horizon at the 95% confidence interval.

As we can see from Table 6 to Table 9 the $VaR(a)$ declines monotonically as we move to higher scales. In other words, the $VaR(a)$ is higher at lower scales. Similarly, the contribution of the $VaR(a)$ is higher at lower scales and decreases as we move to higher scales. A potential loss of the portfolio is higher when we focus on lower scales. Finally, we can observe that the total $VaR(a)$ estimated from the raw data and the total $VaR(a)$ estimated from the recomposed data are very close. Our results are similar to the ones presented in Fernandez (2006) and Masih *et al.* (2010) and suggest that risk is concentrated at the lower scale of the data. In all time samples, scale 1 contributes with more than 42% to the total $VaR(a)$ while in some cases reaches up to 55%. As Maharaj *et al.* (2012) noted, the lower scales capture the activity of speculative traders and the higher scales reflect the sentiments of investor with medium to long-term investment horizons. This finding has important implications for scalpers, day traders and position traders.

A closer inspection of Table 6 reveals that the total $VaR(a)$ is relatively low for all countries. More precisely, the lower values are observed in Portugal and Italy, 0.009 and 0.012 respectively. The higher value is estimated for Greece and it is 0.0164. However, these values are not significantly different than the ones observed in France and Germany, 0.0146 and 0.0136 respectively.

In Table 7 the $VaR(a)$ is estimated during the crisis time-period. A closer inspection of Table 7 reveals that the $VaR(a)$ has grown threefold almost for every

country. Again, the lower values are observed for Portugal and Italy, 0.0259 and 0.0268, respectively while the higher values are observed in France and Netherlands, 0.0354 and 0.0344, respectively. As Bai *et. al.* (2003) noted, extreme returns occur more frequently in crisis period, information is therefore important for all groups of traders, such as hedgers, speculators and arbitragers in both cash and derivative markets. The evidence is also consistent with the findings of Vo (2014).

In Table 8 the $VaR(a)$ estimated in the first post-crisis time period can be found. This period is also the same as when the European crisis started. The effects can be found in the estimation of $VaR(a)$ in Greece, which was further increased. On the contrary the $VaR(a)$ from the remaining countries was decreased or remained stable (Italy).

The results of our analysis between December 1, 2011 and September 10, 2012 are presented in Table 9. During this time Greece was in deep crisis while Spain and Italy were under a rescue plan. This can be reflected from the estimated $VaR(a)$ in each country. For Greece the $VaR(a)$ is 0.0514 while for Spain and Italy is 0.0278 and 0.0309. On the other hand the $VaR(a)$ for Germany, UK, Netherlands and France is 0.0201, 0.017, 0.0226 and 0.0258, respectively. Surprisingly, the estimated $VaR(a)$ for Portugal, another country with financial problems, is 0.0202.

Overall, period specific VaR analysis provides a more detailed breakdown of the market risk compared to the whole period. VaR has grown threefold almost for every country during the period of global financial crisis. $VaRs$ in debt-ridden countries are larger during Eurozone crisis period relatively to pre-global financial crisis period, with the exception of Portugal.

7. Forecasting the Multiscale Nature of Systematic Risk

In this section, the analysis of the multiscale nature of the systematic risk will be further expanded by employing WNs. More precisely, WNs will be used in order to capture and forecast the dynamics and the multiscale nature of the systematic risk.

We will use one period of our dataset for in-sample training and one period for out-of-sample forecasting. In order to do so, the third data set that represents the post-crisis period in US is used to train artificial WNs. Then the trained WNs will be used in order to forecast the betas in the out-of-sample period which is the fourth data set ranging from December 1, 2011 to September 10, 2012 and represents the current situation in Europe.

7.1 Wavelet Neural Networks

WNs are a new class of networks that combine the classic sigmoid neural networks and the WA. WNs have been used with great success in a wide range of applications. For a complete theoretical background and a concise treatment of WNs, readers are referred to Alexandridis and Zapranis (2014).

A WN usually has the form of a three layer network. In the input layer the explanatory variables are introduced to the WN. The hidden layer consists of the hidden units (HUs). In the hidden layer the input variables are transformed to dilated and translated version of the mother wavelet. Finally, in the output layer, the approximation of the target values is estimated. The structure of a single hidden-layer feed forward WN is given in Figure 3. The network output is given by the following expression:

$$g_{\lambda}(\mathbf{x}; \mathbf{w}) = \hat{y}(\mathbf{x}) = w_{\lambda+1}^{[2]} + \sum_{j=1}^{\lambda} w_j^{[2]} \cdot \Psi_j(\mathbf{x}) + \sum_{i=1}^m w_i^{[0]} \cdot x_i . \quad (28)$$

In that expression, $\Psi_j(\mathbf{x})$ is a multidimensional wavelet which is constructed by the product of m scalar wavelets, \mathbf{x} is the input vector, m is the number of network inputs, λ is the number of HUs and w stands for a network weight.

7.2 Training and Forecasting

First, the WNs, $g_\lambda(\mathbf{x}; \mathbf{w})$, had to be trained. For each stock, wavelet networks were trained for each scale for both the detail D_j and smooth components S_j . In order to determine the lag series of the training patterns and the network topology, i.e. the number of the HUs, the variable selection algorithm described in Alexandridis and Zapranis (2014) was followed. The WNs were trained using the data from the first post-crisis period.

In order to evaluate the performance of the WNs in predicting the dynamics of the multiscale betas the 1-period-ahead forecasting method has been employed. More precisely the WNs were trained on the decomposed data of the first post-crisis period in order to forecast the values of the beta on the second post-crisis period. Note, that the data from the second post-crisis period have not been used for training or calibration of the WNs. Hence, we produced recursively using a rolling window of 203 out-of-sample one-period-ahead forecasts.

In Table 10 the forecasted values of the average betas and R^2 from each market are presented for each scale as well as the raw values. The standard deviation, the skewness and the kurtosis are also reported. Comparing the forecasted values from Table 10 against the estimated ones presented in Table 5 we can conclude that the WN has the ability to accurately forecast both the betas and the R^2 . The multi resolution analysis of the systematic risk allowed the WN to be efficiently trained. As a result the WN captured the dynamics of the systematic risk and it was able to accurately forecast them. Next, comparing the real and forecasted VaR at different timescales in Table 11 and Table 9 we can observe that the WN slightly underestimates the VaR for all countries. However, the basic dynamics of VaR were successfully captured as well as the changes of the VaR according to scale.

Finally, in Figure 4, the 1-period ahead forecast of excess returns of the AEX index is presented. The forecasts were based on the multiscale analysis of the WA. Then, WNs were used to learn the dynamics of the returns in each scale. The trained networks were used to forecast the excess returns. A closer inspection of Figure 4 reveals that the proposed method can accurately track the excess returns. More analytically, the normalized mean square error is 0.148 while the mean and maximum absolute error is only 0.002 and 0.034 respectively.

Due to space limitations we present the results only for the AEX index. The results for the remaining countries and stocks are similar and are available upon request from the authors.

8. Conclusions

The US subprime loan crisis unleashed a series of negative effects on the global economy ranging from the stock market collapse, financial institutions failure and global recession. The meltdown of the subprime crisis of 2007 exerted a meteor shower effect across the world's stock market by the fourth quarter of 2008. In the last quarter of 2008, the stock markets of both developed and emerging economies experienced large decline in prices of securities. In this paper, we have investigated the impact of the global financial crisis on the systematic and market risks in eight European markets: France, Germany, Greece, Italy, Netherlands, Portugal, Spain and the UK using the framework of a capital asset pricing model.

In our analysis we first investigated whether the U.S. crisis affected the European stock markets by studying the relationship between the U.S market and the eight different European countries. Our results indicate that the correlation between the markets increased during the crisis period but significantly decreased when the U.S.

market started to recover, and correlation increased again when the crisis moved to the Eurozone.

Next, we have studied the multiscale systematic risk locally by applying a national CAPM. Our empirical results indicate that average beta coefficients have a multiscale tendency and betas tend to increase at mid to higher scales for the whole period supporting the CAPM at medium time horizons. During the sub period of financial crisis, the size of betas tends to increase for some countries and R^2 s increase for every country relatively to the pre-crisis period. The increase of multi-scale betas during periods of global financial crisis and Eurozone crisis has been induced by a combination of leverage effect and asymmetric response of the market to bad news. The evidence of leverage effect accords well with the findings of Iqbal and Kume (2014) in their recent study of markets from the UK, France and Germany and the evidence of asymmetric effect in time-varying beta is consistent with the recent findings of Choudhry and Jayasekera (2012, 2013, 2014). Moreover, in our analysis, the results from the two post-crisis samples, indicate that changes of both betas and R^2 varies between the two groups of the European markets.

The scale dependent *VaR* results suggest that risk is concentrated at the lower scale of the data. *VaR* estimates tend to increase threefold almost for every country during the global financial crisis period relative to the pre-crisis period. Therefore, a potential loss of portfolio is higher at lower scales; furthermore, a potential loss of portfolio across scales is far higher during the crisis period. The evidence of multiscale nature of systematic risk and market risk has important policy implication for financial practitioners, fund managers, researchers and policy-makers.

Finally, WNs were employed in order to capture the dynamics of the multiscale systematic risk. Our results indicate that WNs can accurately forecast both the betas and the *VaRs*.

References

- Alexandridis, A., and Zapranis, A. (2014). *Wavelet Networks: Methodologies and Applications in Financial Engineering, Classification and Chaos*. New Jersey, USA: Wiley.
- Bae, K. H., Karolyi, G. A., and Stultz, R. M., 2003. A new approach to measuring financial contagion. *Review of Financial Studies* 16 (3), 717-763.
- Ball, R., and Kothari, S. P., 1989. Nonstationary expected returns: implications for tests of market efficiency and serial correlation in returns. *Journal of Financial Economics* 25, 51-74.
- Banti, C., Phylaktis, K., and Sarno, L., 2012. Global liquidity risk in the foreign exchange market. *Journal of International Money and Finance* 31, 267-291.
- Bartram, S. M., Bodnar, G., 2009. No place to hide: the global crisis in equity markets in 2008-2009. *Journal of International Money and Finance* 28, 1246-1292.
- Beckaert, K., Finnerty, J., and Gupta, M., 1990. The intertemporal relations between the US and Japanese stock markets. *Journal of Finance* 45, 1297-1306.
- Black, F., 1976. Studies of stock market volatility changes. In: *Proceedings of the American Statistical Association, Business and Economics Statistics Section*, pp. 177-181.
- Bolerslev, T., Engle, R. F., and Wooldridge, J., 1988. A capital asset pricing model with time-varying covariances. *Journal of Political Economy* 96, 116-131.

- Brailsford, T. J., and Faff, R. W., 1997. Testing the conditional CAPM and the effect of intervaling: a note. *Pacific-Basin Finance Journal* 5, 527-537.
- Braun, R., Nelson, D., and Sunier, A., 1995. Good news, bad news, volatility and betas. *Journal of Finance* 50, 1575-1601.
- Calomiris, C. W., 2009. Financial innovation, regulation and reform. *Cato Journal* 29 (1), 65-91.
- Chiang, T. C., Jeon, B. N., and Li, H., 2007. Dynamic correlation analysis of financial contagion: Evidence from Asian market. *Journal of International Money and Finance* 26 (7), 1206-1228.
- Cho, Y. H., and Engle, R. F., 1999. *Time-varying Betas and Asymmetric Effects of News: Empirical Analysis of Blue Chips Stock*. National Bureau of Economic Research Working Paper (7330).
- Choudhry, T., Lu, L., and Peng, K. (2007) Common stochastic trends among Far East stock prices: effects of the Asian financial crisis. *International Review of Financial Analysis* 16, 242-261.
- Choudhry, T., Jayasekera, R., 2012. Comparison of efficiency characteristics between the banking sectors of US and UK during the global financial crisis of 2007-2011. *International Review of Financial Analysis* 25, 106-116.
- Choudhry, T., Jayasekera, R., 2013. Level of efficiency in the UK equity market: empirical study of the effects of the global financial crisis. *Review of Quantitative Finance and Accounting*, DOI 10.1007/s11156-013-0404-6
- Choudhry, T., and Jayasekera, R., 2014. Market efficiency during the global financial crisis: evidence from European banks. *Journal of International Money and Finance*, <http://dx.doi.org/10.1016/j.jimonfin.2014.03.008>

- Christie, A., 1982. The stochastic behaviour of common stock variances: value, leverage and interest rate affects. *Journal of Financial Economics* 10, 407-432.
- Classens, S., Forbes, K., 2001. *International and Financial Contagion*. Edited volume, Kluwer, Boston.
- Conner, J., and Rossiter, R., 2005. Wavelet transforms and commodity prices. *Studies in Nonlinear Dynamics & Econometrics* 9, Article 6.
- Dharan, B., and Ikenberry, D., 1995. The long-run negative drift of post-listing stock returns. *Journal of Finance* 50, 1547-1574.
- Donoho, D. L., and Johnstone, I. M., 1994. Ideal spatial adaption by wavelet shrinkage. *Biometrika* 81, 425-455.
- Dorn, A. J., 2009. Editor's note. Cato Institute's 26th Annual Monetary Conference, November 19, 2009 on the 'Financial Market Crisis of 2009', published in the *Cato Journal* 29 (1).
- Dowd, K., 2009. Moral hazard and the financial crisis. *Cato Journal* 29 (1), 141-166.
- Dungey, M., and Martin, V., 2007. Unravelling financial markets linkage during crisis, *Journal of Applied Econometrics* 22, 89-119.
- Dungey, M., and Timbakis, D., 2005. *Identifying International Financial Contagion: Progress and Challenge*. Edited Volume, Oxford University Press, New York.
- Eichengreen, B., Mody, A., Nadelykovic, M., and Sarno, L., 2012. How the subprime crisis went global: evidence from bank credit default swap spreads. *Journal of International Money and Finance*, 31, 1299-1318.
- Engle, R. F., Ito, T., and Lin, W., 1990. Meteor showers or heat wave? Heteroscedastic intra-daily volatility in the foreign exchange market. *Econometrica* 48, 525-542.

- Engle, R. F., Gallo, G. M., and Velucchi, M. 2012., Volatility spillovers in East Asian financial markets: a MEM-based approach. *Review of Economics and Statistics* 94, 222-233.
- Fama, E. F., and French, K. R., 1996. The CAPM is wanted, dead or alive. *The Journal of Finance* 51, 1947-1958.
- Fernandez, V. P., 2005. The international CAPM and a wavelet-based decomposition of value at risk. *Studies in Nonlinear Dynamics and Econometrics* 9(4).
- Fernandez, V., 2006. The CAPM and value at risk at different time-scales. *International Review of Financial Analysis* 15, 203-219.
- Fernandez, V. (2008), 'Multi-period hedge ratios for a multi-asset portfolio when accounting for returns co-movement', *Journal of Futures Markets*, 28(2):182-207.
- Forbes, K., and Rigobon, R., 2002. No contagion, only interdependence : measuring stock market comovement. *Journal of Finance* 57, 2223-2261.
- Frankfurter, G. M., Leung, W. K., and Brockman, P. D., 1994. Compounding period length and the market model. *Journal of Economics and business* 46, 179-193.
- Frazzini, A. (2006) The disposition effect and underreaction to news. *Journal of Finance* 61, 2017-2046.
- Gausden, R., and Whitfield, I. A., 2000. Testing the stochastic implications of the life cycle-permanent income hypothesis using UK regional time-series data. *Applied Economics* 32, 1299-1310.
- Gençay, R., Selçuk, F., and Whitcher, B., 2002. *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*. Academic Press, London.
- Gençay, R., Selçuk, F., and Whitcher, B., 2005. Multiscale systematic risk. *Journal of International Money and Finance* 24, 55-70.

- Glosten, L. R., Jagannathan, R., and Runkle, D. E., 1993. On the relation between expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 48, 1779-1801.
- Handa, P., Kothari, S. P., and Wasley, C., 1993. Sensitivity of multivariate tests of the Capital Asset-Pricing Model to the return measurement interval. *The Journal of Finance* 48, 1543-1551.
- In, F., and Kim, S., 2006. The hedge ratio and the empirical relationship between the stock and futures markets: a new approach using wavelet analysis. *Journal of Business* 79, 799-820.
- In, F., and Kim, S., 2007. A note on the relationship between Fama–French risk factors and innovations of ICAPM state variables. *Finance Research Letters* 4, 165-171.
- Iqbal, A., and Kume, O., 2014. Impact of the Financial Crisis on Firms' Capital Structure in the UK, France and Germany. *Multinational Finance Journal* (in press).
- Kim, S., and In, F., 2005. The relationship between stock returns and inflation: new evidence from wavelet analysis. *Journal of Empirical Finance* 12, 435– 444.
- Kim, S., and In, F., 2007. On the relationship between changes in stock prices and bond yields in the G7 countries: Wavelet analysis. *Journal of International Financial Markets Money and Finance* 17, 167-179.
- Kothari, S. P., Shanken, J. A. Y., and Sloan, R. G., 1995. Another look at the cross-section of expected stock returns. *Journal of Finance* 50, 185-224.
- Lin, W., Engle, R. F., and Ito, T., 1994. Do bulls and bears move across borders? International transmission of stock returns and volatility. *Review of Financial Studies* 7, 507-538.

- Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The Review of Economics and Statistics* 47, 13-37.
- Loughran, T., and Ritter, J., 1995. The new issues puzzle. *Journal of Finance* 50, 23-51.
- Maharaj, E. A., Galagedera, D. U. A., and Dark, J., 2011. A comparison of developed and emerging equity market return volatility at different time scales. *Managerial Finance* 37(10), 940-952.
- Masih, A. M M., and Masih, R., 2002. Propagative causal price transmission among international stock markets: evidence from the pre and post globalization. *Global Finance Journal* 13, 63-91.
- Masih, M., Alzahrani, M., and Al-Titi, O., 2010. Systematic risk and time scales: new evidence from an application of wavelet approach to the emerging Gulf stock markets. *International Review of Financial Analysis* 19, 10-18.
- Norsworthy, J. R., Li, D., and Gorener, R., 2000. Wavelet-based analysis of time series: an export from engineering to finance. *Engineering Management Society*, 2000. *Proceedings of the 2000 IEEE*, 126-132.
- Percival, D., and Walden, A., 2000. *Wavelets Analysis for Time Series Analysis*. Cambridge University Press, Cambridge, U.K.
- Poterba, T. M., and Summers, L., 1986. The persistence of volatility and stock market fluctuation. *American Economic Review* 76, 1142-1151.
- Rabeh, K. K., and Mohamed, B. B., (2011). A time-scale analysis of systematic risk: wavelet-based approach. Report, Munich Personal RePEc Archive, MPRA Paper No. 31938.

- Ramsey, J. B., 1999. The contribution of wavelets to the analysis of economic and financial data. *Philosophical Transactions: Mathematical, Physical and Engineering Sciences* 357(1760), 2593-2606.
- Rua, A., and Nunes, L. C., 2012. A wavelet-based assessment of market risk: the emerging markets case. *The Quarterly Review of Economics and Finance*, 52, 84-92.
- Sharpe, W. F., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. *The Journal of Finance* 19, 425-442.
- Schwartz, A. J., 2009. Origins of the financial market crisis in 2008. *Cato Journal* 29 (1), 19-23.
- Syllignakis, M. N., and Kouretas, G. P., 2011. Dynamic correlation analysis of financial contagion: Evidence from the Central and Eastern European markets. *International Journal of Economics and Finance* 20 (4), 717-732.
- Theodossiou, P. and Lee, U. (1993) "Mean and volatility spillovers between international equity markets: further empirical evidence", *Journal of Financial Research*, vol. 16, pp. 337-350.
- Vardar, G., and Aydogan, B., 2014. *Benefits of International Portfolio Diversification During the Financial Crisis*. Paper presented at the 21st Annual Conference of the Multinational Finance Society during June 28 – July 2, 2014 at Prague.
- Vo, D-V., 2014. *A Coexistence Approach on Financial Contagion*. Paper presented at the 21st Multinational Finance Society Conference during June 28 – 2 July, 2014, Prague.

Zapranis, A., and Alexandridis, A., 2008. Modelling temperature time dependent speed of mean reversion in the context of weather derivative pricing. *Applied Mathematical Finance* 15, 355 - 386.

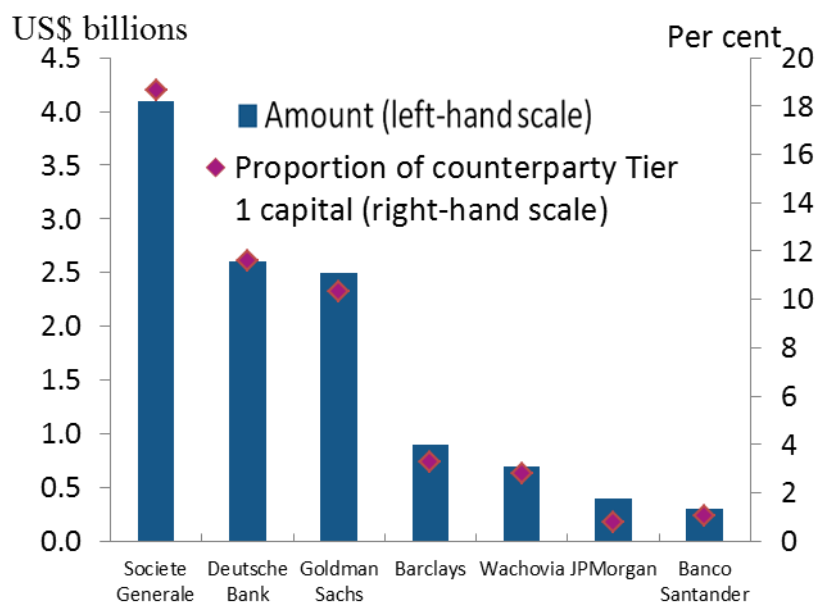


Figure 1: Selected counterparty exposures to AIG at the time of its failure.

Sources: American International Group (d) and Capital IQ.

(a) The chart shows collateral that AIG returned between 16 September and 31 December 2008 to retire CDS obligations which existed at the time of its failure.

(b) Selected counterparties shown. Does not represent total exposure to AIG.

(c) Tier 1 capital as of 30 June 2008 as reported in each bank's accounts. Goldman Sachs data are for 29 August 2008.

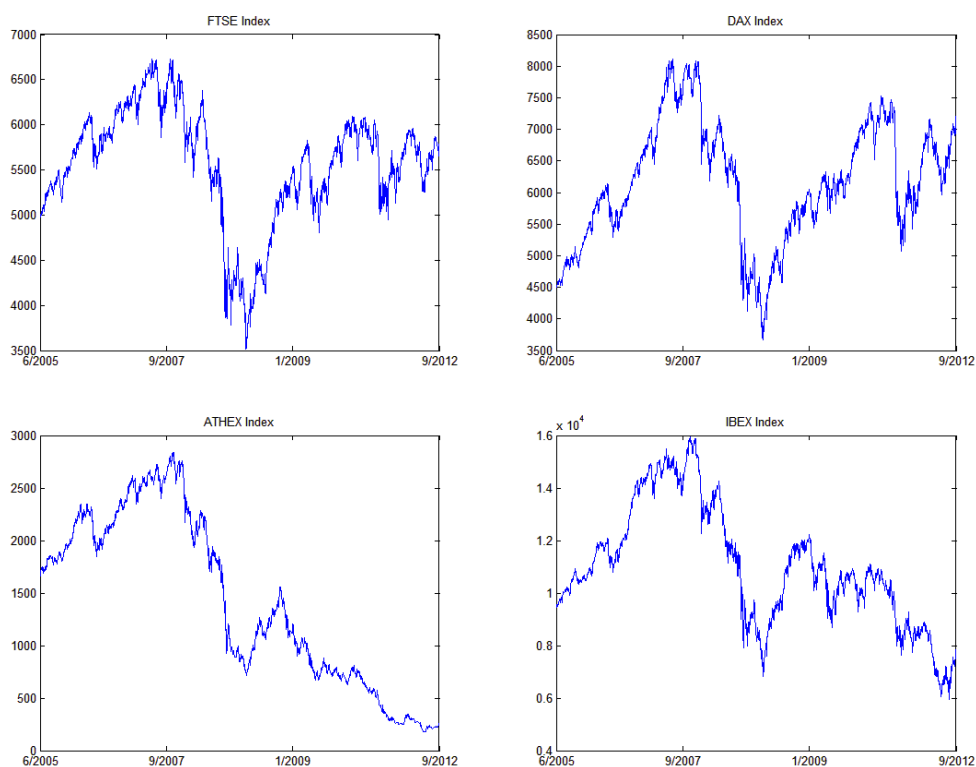


Figure 2. Temporal movements of stock indices for the markets of the UK, Germany, Greece, and Spain

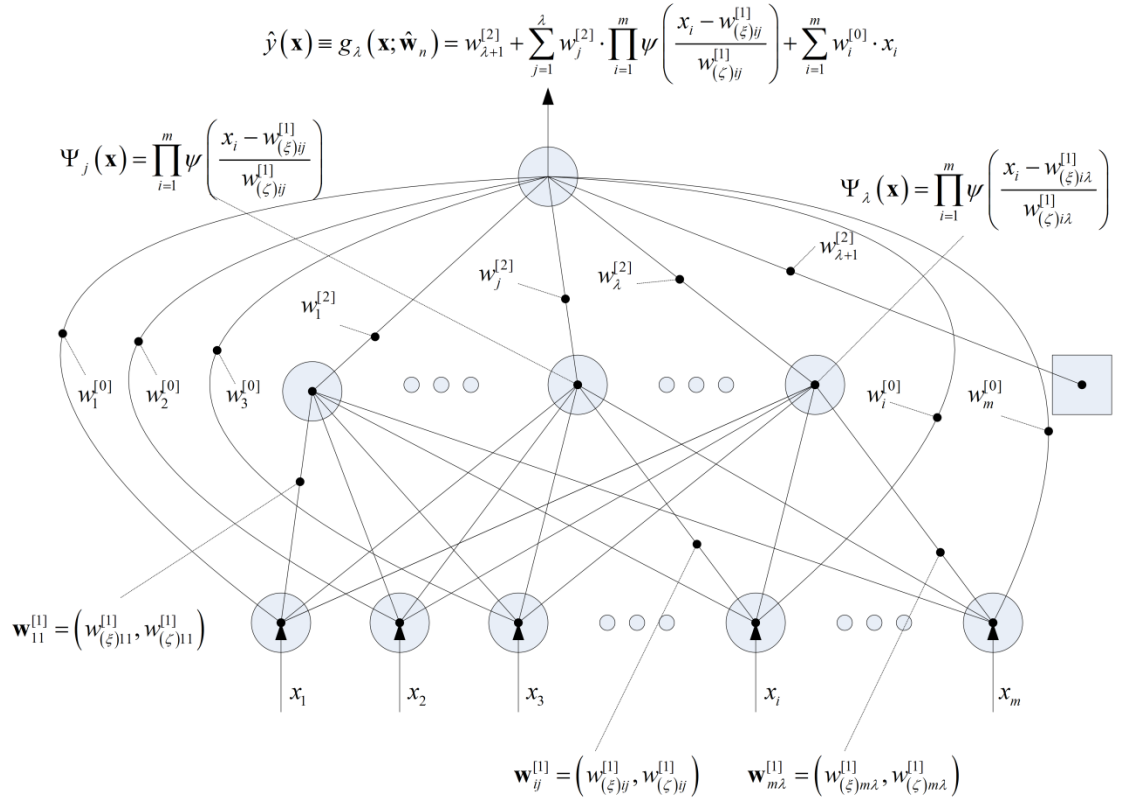


Figure 3. A Feed forward Wavelet Network.

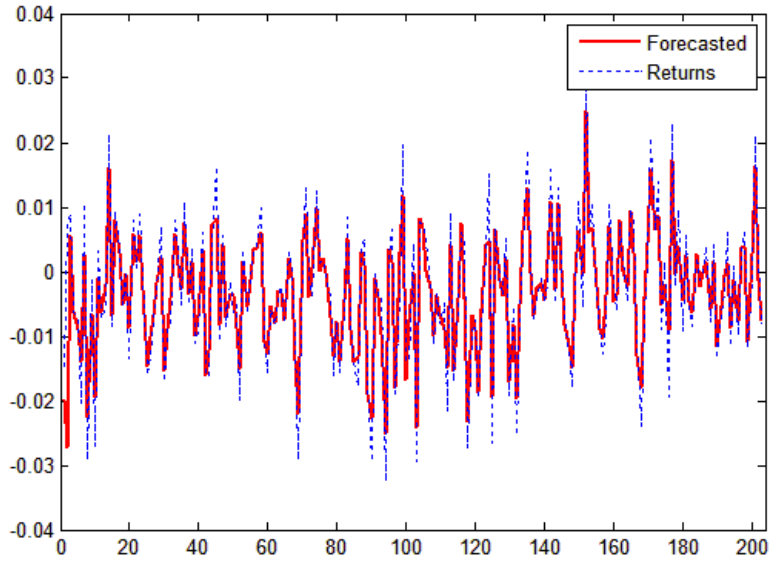


Figure 4. The real and forecast of excess returns of the AEX index

Table 1. Correlation between the S&P 500 and the European stock markets in different time-periods.

| | | AEX | ATHEX | CAC 40 | DAX 30 | FTSE 100 | IEX 35 | MIB | PSI-20 |
|-------------|---------|------|-------|--------|--------|----------|--------|-------|--------|
| Pre | S&P 500 | 0.96 | 0.94 | 0.97 | 0.98 | 0.95 | 0.96 | 0.96 | 0.96 |
| In | | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 | 0.97 | 0.98 | 0.95 |
| First Post | | 0.50 | -0.50 | 0.32 | 0.83 | 0.87 | -0.14 | -0.06 | -0.11 |
| Second Post | | 0.61 | 0.11 | 0.66 | 0.92 | 0.62 | -0.26 | 0.16 | -0.11 |
| Whole | | 0.81 | 0.45 | 0.71 | 0.83 | 0.95 | 0.55 | 0.57 | 0.57 |

Table 2. Beta and R² computed from recomposed crystals of each index. Pre crisis period

| | Beta at each scale | | | | | R ² at each scale | | | | | Raw Data | |
|-----------------|--------------------|-------|-------|------|-------|------------------------------|------|-------|-------|-------|----------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R ² |
| AEX | | | | | | | | | | | | |
| Mean | 0.93 | 0.95 | 0.95 | 0.98 | 0.91 | 0.29 | 0.28 | 0.34 | 0.36 | 0.31 | 0.95 | 0.31 |
| SD | 0.23 | 0.25 | 0.25 | 0.27 | 0.34 | 0.16 | 0.16 | 0.16 | 0.17 | 0.17 | 0.20 | 0.15 |
| Skew. | 0.23 | 0.25 | 0.31 | 1.07 | 0.26 | 1.61 | 1.49 | 1.32 | 0.45 | 0.42 | 0.15 | 1.66 |
| Kurtosis | 2.71 | 1.80 | 2.15 | 3.48 | 3.57 | 4.42 | 4.41 | 4.30 | 2.86 | 3.07 | 2.32 | 4.87 |
| ATHEX | | | | | | | | | | | | |
| Mean | 0.89 | 0.82 | 0.92 | 0.98 | 0.92 | 0.28 | 0.27 | 0.33 | 0.38 | 0.32 | 0.89 | 0.30 |
| SD | 0.32 | 0.26 | 0.27 | 0.27 | 0.46 | 0.18 | 0.17 | 0.18 | 0.18 | 0.23 | 0.26 | 0.17 |
| Skew. | -0.25 | 0.40 | 0.08 | 0.06 | -0.01 | 0.75 | 0.91 | 0.57 | 0.62 | 0.44 | -0.01 | 0.79 |
| Kurtosis | 2.51 | 3.75 | 2.31 | 1.96 | 1.94 | 3.44 | 3.48 | 3.06 | 3.01 | 2.45 | 2.48 | 3.44 |
| CAC 40 | | | | | | | | | | | | |
| Mean | 1.01 | 1.00 | 1.01 | 1.06 | 1.04 | 0.39 | 0.36 | 0.39 | 0.42 | 0.37 | 1.01 | 0.40 |
| SD | 0.24 | 0.22 | 0.25 | 0.32 | 0.39 | 0.15 | 0.14 | 0.16 | 0.16 | 0.17 | 0.22 | 0.14 |
| Skew. | 0.28 | -0.17 | -0.18 | 1.12 | 0.34 | 0.67 | 0.35 | 0.45 | 0.33 | -0.09 | 0.17 | 0.67 |
| Kurtosis | 2.70 | 2.75 | 2.64 | 4.97 | 4.24 | 2.71 | 2.59 | 2.44 | 2.02 | 2.26 | 2.74 | 2.68 |
| DAX 30 | | | | | | | | | | | | |
| Mean | 0.84 | 0.88 | 0.91 | 0.98 | 0.98 | 0.31 | 0.33 | 0.41 | 0.43 | 0.40 | 0.88 | 0.35 |
| SD | 0.18 | 0.17 | 0.24 | 0.26 | 0.29 | 0.12 | 0.13 | 0.16 | 0.14 | 0.17 | 0.17 | 0.12 |
| Skew. | 0.23 | -0.06 | 0.04 | 0.41 | 0.84 | 0.82 | 0.96 | 0.23 | 0.15 | 0.25 | 0.22 | 0.85 |
| Kurtosis | 2.32 | 2.10 | 2.30 | 2.61 | 4.34 | 3.73 | 4.19 | 2.43 | 2.26 | 2.07 | 2.10 | 3.61 |
| FTSE 100 | | | | | | | | | | | | |
| Mean | 1.03 | 1.03 | 1.11 | 1.11 | 0.97 | 0.31 | 0.29 | 0.34 | 0.38 | 0.33 | 1.05 | 0.32 |
| SD | 0.38 | 0.35 | 0.45 | 0.41 | 0.46 | 0.12 | 0.11 | 0.14 | 0.15 | 0.18 | 0.35 | 0.11 |
| Skew. | 1.26 | 1.15 | 1.05 | 0.46 | 0.89 | 0.29 | 0.26 | 0.11 | -0.03 | 0.26 | 1.14 | 0.13 |
| Kurtosis | 4.65 | 4.15 | 4.13 | 2.76 | 3.51 | 2.38 | 2.41 | 2.11 | 2.41 | 2.32 | 4.17 | 2.13 |
| IBEX 35 | | | | | | | | | | | | |
| Mean | 1.02 | 1.04 | 1.08 | 1.18 | 1.13 | 0.41 | 0.39 | 0.42 | 0.48 | 0.50 | 1.05 | 0.42 |
| SD | 0.23 | 0.24 | 0.34 | 0.43 | 0.38 | 0.16 | 0.15 | 0.16 | 0.15 | 0.16 | 0.24 | 0.15 |
| Skew. | 0.41 | 0.46 | 0.46 | 1.15 | 0.85 | 0.63 | 0.52 | -0.11 | -0.15 | -0.16 | 0.66 | 0.58 |
| Kurtosis | 2.25 | 2.68 | 3.26 | 3.64 | 3.02 | 2.77 | 3.02 | 2.61 | 2.74 | 2.49 | 2.79 | 2.81 |
| MIB | | | | | | | | | | | | |
| Mean | 0.89 | 0.92 | 0.97 | 0.99 | 0.99 | 0.30 | 0.26 | 0.31 | 0.34 | 0.37 | 0.93 | 0.32 |
| SD | 0.31 | 0.28 | 0.33 | 0.35 | 0.33 | 0.15 | 0.12 | 0.14 | 0.15 | 0.16 | 0.28 | 0.13 |
| Skew. | -0.22 | -0.59 | 0.20 | 0.71 | 0.00 | 0.28 | 0.26 | 0.47 | 0.19 | -0.20 | -0.29 | 0.28 |
| Kurtosis | 2.92 | 2.85 | 2.37 | 4.01 | 2.54 | 2.73 | 2.60 | 2.93 | 2.00 | 2.35 | 2.57 | 2.81 |
| PSI-20 | | | | | | | | | | | | |
| Mean | 0.73 | 0.74 | 0.81 | 0.97 | 0.96 | 0.13 | 0.12 | 0.17 | 0.27 | 0.23 | 0.81 | 0.16 |
| SD | 0.41 | 0.49 | 0.36 | 0.39 | 0.54 | 0.12 | 0.12 | 0.14 | 0.15 | 0.16 | 0.35 | 0.12 |
| Skew. | -0.12 | 0.03 | -0.38 | 0.80 | 0.24 | 1.15 | 1.17 | 1.05 | -0.10 | 0.18 | -0.44 | 0.93 |
| Kurtosis | 2.57 | 2.01 | 3.80 | 2.49 | 1.99 | 2.81 | 3.02 | 3.20 | 1.63 | 1.89 | 2.77 | 2.56 |

Table 3. Beta and R² computed from recomposed crystals of each index. In crisis period

| | Beta at each scale | | | | | R ² at each scale | | | | | Raw Data | |
|-----------------|--------------------|-------|-------|-------|-------|------------------------------|-------|-------|-------|-------|----------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R ² |
| AEX | | | | | | | | | | | | |
| Mean | 0.93 | 0.95 | 1.05 | 1.09 | 1.01 | 0.45 | 0.46 | 0.44 | 0.42 | 0.50 | 0.97 | 0.47 |
| SD | 0.37 | 0.42 | 0.49 | 0.56 | 0.41 | 0.13 | 0.15 | 0.15 | 0.17 | 0.13 | 0.38 | 0.12 |
| Skew. | 1.21 | 1.01 | 0.51 | 0.40 | 0.74 | 0.43 | -0.18 | -0.62 | -0.47 | -0.19 | 0.98 | 0.05 |
| Kurtosis | 3.76 | 3.63 | 2.72 | 2.23 | 2.54 | 2.74 | 2.68 | 2.66 | 2.07 | 1.86 | 3.35 | 2.79 |
| ATHEX | | | | | | | | | | | | |
| Mean | 0.84 | 0.83 | 0.84 | 0.83 | 0.92 | 0.43 | 0.42 | 0.40 | 0.47 | 0.46 | 0.85 | 0.45 |
| SD | 0.31 | 0.32 | 0.36 | 0.34 | 0.35 | 0.20 | 0.20 | 0.22 | 0.21 | 0.21 | 0.30 | 0.19 |
| Skew. | 0.20 | 0.25 | -0.31 | 0.55 | -0.14 | 0.41 | 0.31 | 0.11 | 0.23 | 0.09 | 0.20 | 0.36 |
| Kurtosis | 2.09 | 2.59 | 2.75 | 3.17 | 1.54 | 1.94 | 2.24 | 2.47 | 2.71 | 2.33 | 2.23 | 2.15 |
| CAC 40 | | | | | | | | | | | | |
| Mean | 1.00 | 1.05 | 1.13 | 1.14 | 1.12 | 0.52 | 0.53 | 0.47 | 0.49 | 0.47 | 1.04 | 0.53 |
| SD | 0.28 | 0.31 | 0.42 | 0.45 | 0.35 | 0.12 | 0.13 | 0.14 | 0.16 | 0.13 | 0.29 | 0.11 |
| Skew. | 0.47 | 0.07 | 0.02 | -0.01 | 0.11 | 0.30 | -0.01 | -0.63 | -0.67 | -0.92 | 0.20 | 0.20 |
| Kurtosis | 2.64 | 2.40 | 2.26 | 2.37 | 3.37 | 2.56 | 2.38 | 2.86 | 2.71 | 4.99 | 2.38 | 2.57 |
| DAX 30 | | | | | | | | | | | | |
| Mean | 0.84 | 0.87 | 1.03 | 0.93 | 0.94 | 0.35 | 0.38 | 0.39 | 0.42 | 0.43 | 0.89 | 0.39 |
| SD | 0.28 | 0.30 | 0.39 | 0.42 | 0.39 | 0.14 | 0.16 | 0.17 | 0.19 | 0.19 | 0.29 | 0.14 |
| Skew. | 0.03 | -0.03 | -0.13 | 0.08 | 0.20 | -0.27 | 0.00 | -0.12 | -0.32 | -0.48 | 0.00 | -0.20 |
| Kurtosis | 2.16 | 2.10 | 2.45 | 2.57 | 2.65 | 2.04 | 2.05 | 2.62 | 2.47 | 2.59 | 2.17 | 2.09 |
| FTSE 100 | | | | | | | | | | | | |
| Mean | 0.97 | 0.99 | 1.05 | 1.08 | 0.99 | 0.39 | 0.41 | 0.35 | 0.35 | 0.38 | 1.00 | 0.41 |
| SD | 0.37 | 0.41 | 0.49 | 0.63 | 0.46 | 0.12 | 0.13 | 0.13 | 0.14 | 0.17 | 0.38 | 0.11 |
| Skew. | 1.12 | 1.12 | 0.77 | 1.63 | 1.21 | 0.22 | 0.11 | -0.12 | -0.03 | 0.13 | 1.09 | 0.21 |
| Kurtosis | 3.42 | 3.64 | 2.96 | 6.97 | 4.00 | 3.10 | 2.60 | 2.73 | 2.60 | 2.50 | 3.45 | 3.10 |
| IBEX 35 | | | | | | | | | | | | |
| Mean | 0.96 | 0.93 | 0.93 | 0.95 | 1.02 | 0.53 | 0.50 | 0.50 | 0.45 | 0.39 | 0.96 | 0.53 |
| SD | 0.25 | 0.30 | 0.32 | 0.36 | 0.37 | 0.16 | 0.19 | 0.19 | 0.19 | 0.20 | 0.26 | 0.16 |
| Skew. | 0.21 | 0.06 | 0.28 | 0.25 | 0.25 | 0.42 | 0.09 | 0.12 | 0.18 | 0.25 | 0.12 | 0.28 |
| Kurtosis | 2.60 | 2.39 | 2.40 | 2.45 | 2.48 | 2.58 | 2.28 | 2.39 | 2.98 | 2.45 | 2.37 | 2.61 |
| MIB | | | | | | | | | | | | |
| Mean | 0.77 | 0.76 | 0.83 | 0.89 | 0.92 | 0.37 | 0.39 | 0.39 | 0.40 | 0.46 | 0.80 | 0.41 |
| SD | 0.31 | 0.32 | 0.38 | 0.35 | 0.36 | 0.18 | 0.18 | 0.19 | 0.17 | 0.17 | 0.30 | 0.17 |
| Skew. | 0.37 | 0.27 | 0.16 | 0.03 | 0.65 | 0.33 | 0.18 | -0.29 | -0.11 | -0.01 | 0.20 | 0.05 |
| Kurtosis | 3.32 | 3.11 | 2.94 | 3.00 | 4.09 | 2.58 | 2.76 | 2.51 | 2.37 | 2.31 | 3.27 | 2.75 |
| PSI-20 | | | | | | | | | | | | |
| Mean | 0.92 | 0.95 | 0.96 | 0.96 | 1.03 | 0.40 | 0.38 | 0.41 | 0.38 | 0.42 | 0.96 | 0.43 |
| SD | 0.24 | 0.26 | 0.28 | 0.28 | 0.31 | 0.12 | 0.13 | 0.11 | 0.11 | 0.11 | 0.22 | 0.10 |
| Skew. | -0.36 | -0.63 | -0.32 | -0.51 | 0.62 | -0.38 | 0.45 | -0.08 | -0.91 | 0.33 | -0.42 | 0.16 |
| Kurtosis | 2.92 | 2.94 | 2.00 | 2.13 | 2.04 | 3.02 | 3.92 | 1.79 | 2.92 | 2.97 | 2.35 | 3.24 |

Table 4. Beta and R² computed from recomposed crystals of each index. First post crisis period

| | Beta at each scale | | | | | R ² at each scale | | | | | Raw Data | |
|-----------------|--------------------|-------|-------|-------|-------|------------------------------|-------|-------|-------|-------|----------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R ² |
| AEX | | | | | | | | | | | | |
| Mean | 1.02 | 1.04 | 1.12 | 1.09 | 1.13 | 0.47 | 0.54 | 0.59 | 0.52 | 0.60 | 1.05 | 0.51 |
| SD | 0.44 | 0.42 | 0.45 | 0.44 | 0.41 | 0.15 | 0.14 | 0.16 | 0.17 | 0.15 | 0.42 | 0.14 |
| Skew. | 0.79 | 0.92 | 0.16 | 0.16 | -0.11 | -0.06 | -0.33 | -0.76 | -0.49 | -0.67 | 0.60 | -0.34 |
| Kurtosis | 3.50 | 3.90 | 2.46 | 2.44 | 2.15 | 2.19 | 2.23 | 2.28 | 2.08 | 3.04 | 3.15 | 2.21 |
| ATHEX | | | | | | | | | | | | |
| Mean | 0.86 | 0.88 | 0.87 | 0.94 | 0.89 | 0.46 | 0.51 | 0.49 | 0.46 | 0.45 | 0.87 | 0.48 |
| SD | 0.43 | 0.45 | 0.35 | 0.36 | 0.34 | 0.20 | 0.20 | 0.17 | 0.18 | 0.17 | 0.42 | 0.19 |
| Skew. | 0.58 | 0.65 | 0.24 | 0.19 | 0.39 | 0.32 | 0.14 | 0.13 | -0.13 | 0.05 | 0.56 | 0.23 |
| Kurtosis | 2.07 | 2.11 | 1.64 | 1.83 | 2.08 | 2.17 | 1.90 | 1.83 | 2.03 | 1.88 | 2.02 | 2.04 |
| CAC 40 | | | | | | | | | | | | |
| Mean | 1.02 | 1.03 | 1.07 | 1.04 | 1.06 | 0.58 | 0.64 | 0.65 | 0.58 | 0.61 | 1.04 | 0.61 |
| SD | 0.31 | 0.33 | 0.37 | 0.34 | 0.37 | 0.13 | 0.12 | 0.13 | 0.15 | 0.16 | 0.32 | 0.13 |
| Skew. | 0.47 | 0.73 | 0.44 | 0.43 | 0.27 | -0.44 | -0.32 | -0.75 | -0.40 | -0.51 | 0.52 | -0.45 |
| Kurtosis | 3.13 | 3.26 | 2.94 | 3.18 | 2.43 | 2.65 | 2.52 | 3.38 | 2.29 | 2.58 | 3.06 | 2.79 |
| DAX 30 | | | | | | | | | | | | |
| Mean | 0.91 | 0.92 | 0.94 | 0.94 | 0.92 | 0.45 | 0.55 | 0.58 | 0.50 | 0.53 | 0.92 | 0.51 |
| SD | 0.29 | 0.28 | 0.32 | 0.29 | 0.28 | 0.14 | 0.15 | 0.16 | 0.16 | 0.18 | 0.28 | 0.15 |
| Skew. | -0.31 | -0.18 | -0.06 | -0.04 | 0.01 | -0.76 | -0.69 | -0.62 | -0.22 | 0.07 | -0.27 | -0.67 |
| Kurtosis | 2.29 | 2.65 | 2.12 | 2.21 | 1.90 | 2.88 | 3.11 | 2.26 | 2.37 | 1.93 | 2.23 | 2.76 |
| FTSE 100 | | | | | | | | | | | | |
| Mean | 1.00 | 1.04 | 1.07 | 1.03 | 1.03 | 0.41 | 0.48 | 0.50 | 0.44 | 0.49 | 1.03 | 0.44 |
| SD | 0.43 | 0.39 | 0.49 | 0.47 | 0.52 | 0.15 | 0.14 | 0.17 | 0.18 | 0.21 | 0.42 | 0.15 |
| Skew. | 0.87 | 0.65 | 0.75 | 0.58 | 0.88 | 0.35 | -0.17 | -0.24 | -0.05 | -0.26 | 0.75 | 0.09 |
| Kurtosis | 3.14 | 2.67 | 2.79 | 2.74 | 3.64 | 2.48 | 2.38 | 2.18 | 2.31 | 2.20 | 2.84 | 2.31 |
| IBEX 35 | | | | | | | | | | | | |
| Mean | 0.86 | 0.89 | 0.96 | 0.94 | 1.01 | 0.53 | 0.62 | 0.64 | 0.57 | 0.60 | 0.89 | 0.57 |
| SD | 0.23 | 0.23 | 0.26 | 0.27 | 0.26 | 0.19 | 0.16 | 0.14 | 0.17 | 0.17 | 0.23 | 0.17 |
| Skew. | 0.43 | 0.35 | 0.28 | 0.15 | 0.31 | 0.69 | -0.05 | 0.14 | 0.04 | -0.27 | 0.33 | 0.45 |
| Kurtosis | 2.73 | 2.83 | 1.96 | 2.30 | 2.88 | 2.75 | 3.35 | 2.72 | 2.06 | 2.26 | 2.55 | 2.86 |
| MIB | | | | | | | | | | | | |
| Mean | 0.89 | 0.89 | 0.92 | 0.89 | 0.93 | 0.51 | 0.56 | 0.58 | 0.52 | 0.49 | 0.89 | 0.53 |
| SD | 0.30 | 0.30 | 0.31 | 0.28 | 0.37 | 0.17 | 0.17 | 0.17 | 0.18 | 0.20 | 0.30 | 0.17 |
| Skew. | 0.33 | 0.32 | 0.19 | 0.10 | 0.36 | -0.21 | -0.27 | -0.37 | -0.08 | -0.39 | 0.30 | -0.20 |
| Kurtosis | 3.02 | 3.07 | 2.73 | 2.56 | 2.71 | 2.50 | 2.18 | 2.05 | 2.10 | 2.05 | 2.98 | 2.35 |
| PSI-20 | | | | | | | | | | | | |
| Mean | 0.92 | 0.94 | 0.92 | 0.94 | 1.01 | 0.44 | 0.51 | 0.50 | 0.48 | 0.52 | 0.94 | 0.47 |
| SD | 0.31 | 0.35 | 0.33 | 0.32 | 0.31 | 0.14 | 0.16 | 0.15 | 0.17 | 0.13 | 0.31 | 0.14 |
| Skew. | -0.91 | -0.59 | -1.27 | -0.93 | 0.36 | -1.36 | -1.77 | -1.97 | -1.40 | -0.74 | -0.93 | -1.90 |
| Kurtosis | 4.01 | 3.67 | 4.44 | 3.96 | 1.91 | 5.52 | 6.99 | 7.20 | 4.66 | 2.78 | 4.09 | 7.53 |

Table 5. Beta and R² computed from recomposed crystals of each index. Second post crisis period

| | Beta at each scale | | | | | R ² at each scale | | | | | Raw Data | |
|----------|--------------------|------|-------|------|------|------------------------------|-------|-------|-------|-------|----------|----------------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R ² |
| AEX | | | | | | | | | | | | |
| Mean | 1.23 | 1.21 | 1.24 | 1.26 | 1.20 | 0.43 | 0.43 | 0.45 | 0.36 | 0.26 | 1.22 | 0.42 |
| SD | 0.59 | 0.58 | 0.61 | 0.79 | 0.93 | 0.19 | 0.19 | 0.20 | 0.21 | 0.23 | 0.57 | 0.18 |
| Skew. | 0.21 | 0.39 | 0.58 | 1.14 | 0.06 | -0.45 | -0.36 | -0.22 | 0.24 | 0.59 | 0.31 | -0.46 |
| Kurtosis | 1.79 | 1.98 | 2.15 | 3.68 | 2.06 | 2.83 | 2.47 | 2.37 | 2.03 | 2.42 | 1.89 | 2.68 |
| ATHEX | | | | | | | | | | | | |
| Mean | 0.96 | 1.01 | 1.05 | 1.17 | 1.09 | 0.41 | 0.44 | 0.42 | 0.47 | 0.69 | 1.01 | 0.43 |
| SD | 0.57 | 0.59 | 0.58 | 0.76 | 0.44 | 0.19 | 0.19 | 0.19 | 0.25 | 0.17 | 0.56 | 0.17 |
| Skew. | 1.03 | 0.80 | 0.36 | 0.64 | 0.06 | 0.44 | 0.46 | -0.22 | -0.29 | -1.37 | 0.89 | 0.45 |
| Kurtosis | 2.74 | 2.53 | 2.16 | 2.68 | 2.90 | 2.21 | 2.16 | 2.42 | 1.93 | 5.26 | 2.68 | 2.17 |
| CAC 40 | | | | | | | | | | | | |
| Mean | 1.10 | 1.14 | 1.16 | 1.11 | 1.16 | 0.53 | 0.57 | 0.56 | 0.47 | 0.38 | 1.12 | 0.54 |
| SD | 0.37 | 0.39 | 0.47 | 0.58 | 0.67 | 0.15 | 0.15 | 0.17 | 0.20 | 0.21 | 0.39 | 0.15 |
| Skew. | 0.29 | 0.38 | 0.58 | 1.13 | 0.31 | -0.21 | -0.84 | -0.70 | -0.20 | -0.33 | 0.44 | -0.37 |
| Kurtosis | 2.17 | 2.38 | 2.84 | 4.55 | 2.47 | 2.00 | 3.54 | 2.92 | 1.92 | 1.99 | 2.54 | 2.20 |
| DAX 30 | | | | | | | | | | | | |
| Mean | 0.88 | 0.97 | 0.96 | 0.94 | 1.02 | 0.42 | 0.53 | 0.54 | 0.41 | 0.37 | 0.92 | 0.47 |
| SD | 0.34 | 0.34 | 0.35 | 0.39 | 0.48 | 0.18 | 0.15 | 0.18 | 0.18 | 0.16 | 0.33 | 0.16 |
| Skew. | 0.50 | 0.34 | -0.18 | 0.20 | 0.52 | -0.09 | -0.08 | -0.47 | 0.23 | -0.34 | 0.17 | -0.17 |
| Kurtosis | 3.36 | 2.49 | 2.29 | 2.18 | 2.44 | 2.22 | 2.48 | 2.42 | 1.99 | 2.25 | 2.66 | 2.29 |
| FTSE 100 | | | | | | | | | | | | |
| Mean | 1.07 | 1.17 | 1.18 | 1.10 | 1.00 | 0.41 | 0.40 | 0.39 | 0.36 | 0.30 | 1.10 | 0.40 |
| SD | 0.47 | 0.53 | 0.61 | 0.63 | 0.66 | 0.16 | 0.16 | 0.18 | 0.19 | 0.20 | 0.49 | 0.15 |
| Skew. | 0.68 | 0.44 | 0.37 | 0.75 | 0.75 | -0.17 | -0.13 | -0.25 | 0.13 | 0.25 | 0.55 | -0.10 |
| Kurtosis | 3.08 | 2.41 | 2.52 | 2.88 | 3.26 | 2.20 | 2.23 | 2.23 | 2.05 | 1.99 | 2.65 | 2.17 |
| IBEX 35 | | | | | | | | | | | | |
| Mean | 0.90 | 0.87 | 0.95 | 0.86 | 0.91 | 0.47 | 0.51 | 0.54 | 0.42 | 0.48 | 0.90 | 0.49 |
| SD | 0.31 | 0.31 | 0.35 | 0.42 | 0.56 | 0.21 | 0.21 | 0.20 | 0.24 | 0.26 | 0.31 | 0.20 |
| Skew. | -0.25 | 0.21 | 0.61 | 0.20 | 0.98 | 0.07 | 0.18 | -0.09 | 0.31 | -0.14 | -0.05 | 0.14 |
| Kurtosis | 2.26 | 1.75 | 3.91 | 1.89 | 4.48 | 2.94 | 2.79 | 2.74 | 2.24 | 1.96 | 2.13 | 2.94 |
| MIB | | | | | | | | | | | | |
| Mean | 0.97 | 0.99 | 1.00 | 0.92 | 1.01 | 0.50 | 0.52 | 0.53 | 0.46 | 0.42 | 0.98 | 0.50 |
| SD | 0.38 | 0.42 | 0.49 | 0.44 | 0.64 | 0.16 | 0.16 | 0.19 | 0.20 | 0.25 | 0.41 | 0.16 |
| Skew. | 0.63 | 0.55 | 0.60 | 0.58 | 0.43 | -0.09 | -0.16 | -0.45 | -0.33 | -0.12 | 0.62 | -0.03 |
| Kurtosis | 2.47 | 2.19 | 2.68 | 4.03 | 2.30 | 2.36 | 2.28 | 2.47 | 2.45 | 1.59 | 2.39 | 2.30 |
| PSI-20 | | | | | | | | | | | | |
| Mean | 0.97 | 0.93 | 0.97 | 1.10 | 1.04 | 0.26 | 0.31 | 0.34 | 0.42 | 0.46 | 0.97 | 0.30 |
| SD | 0.51 | 0.49 | 0.51 | 0.73 | 0.69 | 0.15 | 0.16 | 0.17 | 0.19 | 0.28 | 0.51 | 0.15 |
| Skew. | 0.18 | 0.20 | 0.62 | 0.99 | 0.21 | 0.21 | -0.20 | -0.34 | -0.44 | -0.13 | 0.36 | -0.16 |
| Kurtosis | 2.10 | 2.23 | 3.45 | 3.50 | 1.86 | 2.05 | 1.96 | 2.19 | 2.75 | 1.75 | 2.59 | 2.07 |

Table 6. Value At Risk (VaR) at different time scales for equally weighted portfolio. Pre crisis period.

| | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR |
|-----------|--------|------------------------|--------|------------------------|--------|------------------------|--------|------------------------|
| | AEX | | ATHEX | | CAC | | DAX | |
| Scale1 | 0.0091 | 50.14% | 0.0112 | 46.49% | 0.0106 | 53.33% | 0.0095 | 48.50% |
| Scale2 | 0.0062 | 22.84% | 0.0081 | 24.14% | 0.0070 | 23.05% | 0.0066 | 23.73% |
| Scale3 | 0.0051 | 15.54% | 0.0068 | 17.37% | 0.0053 | 13.38% | 0.0054 | 15.94% |
| Scale4 | 0.0037 | 8.24% | 0.0049 | 9.03% | 0.0040 | 7.37% | 0.0040 | 8.55% |
| Scale5 | 0.0023 | 3.24% | 0.0028 | 2.96% | 0.0025 | 2.87% | 0.0025 | 3.29% |
| Total | 0.0129 | | 0.0164 | | 0.0146 | | 0.0136 | |
| Total Raw | 0.0135 | | 0.0171 | | 0.0152 | | 0.0142 | |
| | FTSE | | IBEX | | MIB | | PSI | |
| Scale1 | 0.0093 | 52.02% | 0.0101 | 49.61% | 0.0090 | 54.48% | 0.0060 | 43.56% |
| Scale2 | 0.0061 | 22.28% | 0.0070 | 23.85% | 0.0057 | 21.72% | 0.0045 | 24.42% |
| Scale3 | 0.0049 | 14.79% | 0.0052 | 13.30% | 0.0044 | 13.07% | 0.0036 | 15.43% |
| Scale4 | 0.0036 | 7.98% | 0.0043 | 8.86% | 0.0033 | 7.19% | 0.0031 | 12.00% |
| Scale5 | 0.0022 | 2.93% | 0.0030 | 4.38% | 0.0023 | 3.53% | 0.0019 | 4.60% |
| Total | 0.0128 | | 0.0143 | | 0.0121 | | 0.0091 | |
| Total Raw | 0.0135 | | 0.0149 | | 0.0129 | | 0.0099 | |

Table 7. Value At Risk (VaR) at different time scales for equally weighted portfolio. In crisis period.

| | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR |
|-----------|--------|------------------------|--------|------------------------|--------|------------------------|--------|------------------------|
| | AEX | | ATHEX | | CAC | | DAX | |
| Scale1 | 0.0246 | 51.00% | 0.0227 | 49.31% | 0.0255 | 51.82% | 0.0204 | 49.04% |
| Scale2 | 0.0180 | 27.46% | 0.0166 | 26.24% | 0.0189 | 28.44% | 0.0152 | 27.32% |
| Scale3 | 0.0125 | 13.12% | 0.0116 | 12.89% | 0.0120 | 11.54% | 0.0109 | 14.01% |
| Scale4 | 0.0081 | 5.58% | 0.0093 | 8.27% | 0.0087 | 6.06% | 0.0075 | 6.59% |
| Scale5 | 0.0058 | 2.85% | 0.0059 | 3.29% | 0.0052 | 2.14% | 0.0051 | 3.05% |
| Total | 0.0344 | | 0.0324 | | 0.0354 | | 0.0292 | |
| Total Raw | 0.0364 | | 0.0348 | | 0.0371 | | 0.0312 | |
| | FTSE | | IBEX | | MIB | | PSI | |
| Scale1 | 0.0226 | 52.26% | 0.0232 | 54.64% | 0.0188 | 49.33% | 0.0179 | 47.75% |
| Scale2 | 0.0168 | 28.85% | 0.0159 | 25.61% | 0.0138 | 26.56% | 0.0132 | 25.87% |
| Scale3 | 0.0105 | 11.25% | 0.0112 | 12.67% | 0.0099 | 13.75% | 0.0104 | 15.98% |
| Scale4 | 0.0073 | 5.53% | 0.0072 | 5.29% | 0.0070 | 6.75% | 0.0068 | 6.95% |
| Scale5 | 0.0045 | 2.11% | 0.0042 | 1.79% | 0.0051 | 3.60% | 0.0048 | 3.45% |
| Total | 0.0312 | | 0.0314 | | 0.0268 | | 0.0259 | |
| Total Raw | 0.0329 | | 0.0333 | | 0.0289 | | 0.0284 | |

Table 8. Value At Risk (VaR) at different time scales for equally weighted portfolio. First post crisis period.

| | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR |
|-----------|--------|------------------------|--------|------------------------|--------|------------------------|--------|------------------------|
| | AEX | | ATHEX | | CAC | | DAX | |
| Scale1 | 0.0157 | 44.45% | 0.0274 | 49.51% | 0.0186 | 46.58% | 0.0150 | 44.47% |
| Scale2 | 0.0126 | 28.40% | 0.0215 | 30.47% | 0.0149 | 29.91% | 0.0124 | 30.39% |
| Scale3 | 0.0097 | 16.78% | 0.0138 | 12.56% | 0.0106 | 15.10% | 0.0092 | 16.63% |
| Scale4 | 0.0060 | 6.44% | 0.0087 | 5.03% | 0.0064 | 5.48% | 0.0053 | 5.56% |
| Scale5 | 0.0047 | 3.93% | 0.0061 | 2.42% | 0.0047 | 2.93% | 0.0039 | 2.95% |
| Total | 0.0236 | | 0.0390 | | 0.0273 | | 0.0225 | |
| Total Raw | 0.0240 | | 0.0396 | | 0.0277 | | 0.0230 | |
| | FTSE | | IBEX | | MIB | | PSI | |
| Scale1 | 0.0136 | 45.08% | 0.0168 | 42.66% | 0.0182 | 47.56% | 0.0150 | 45.77% |
| Scale2 | 0.0111 | 29.87% | 0.0145 | 31.67% | 0.0143 | 29.41% | 0.0124 | 31.11% |
| Scale3 | 0.0081 | 15.98% | 0.0106 | 16.76% | 0.0102 | 14.83% | 0.0082 | 13.62% |
| Scale4 | 0.0049 | 5.77% | 0.0063 | 5.95% | 0.0062 | 5.59% | 0.0054 | 5.92% |
| Scale5 | 0.0037 | 3.30% | 0.0044 | 2.96% | 0.0043 | 2.61% | 0.0042 | 3.59% |
| Total | 0.0203 | | 0.0258 | | 0.0264 | | 0.0222 | |
| Total Raw | 0.0206 | | 0.0262 | | 0.0269 | | 0.0227 | |

Table 9. Value at Risk (VaR) at different time scales for equally weighted portfolio. Second post crisis period.

| | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR |
|-----------|--------|------------------------|--------|------------------------|--------|------------------------|--------|------------------------|
| | AEX | | ATHEX | | CAC | | DAX | |
| Scale1 | 0.0166 | 54.04% | 0.0348 | 45.80% | 0.0183 | 50.50% | 0.0135 | 45.01% |
| Scale2 | 0.0115 | 25.85% | 0.0265 | 26.61% | 0.0139 | 29.03% | 0.0113 | 31.52% |
| Scale3 | 0.0086 | 14.57% | 0.0182 | 12.49% | 0.0099 | 14.75% | 0.0084 | 17.32% |
| Scale4 | 0.0048 | 4.49% | 0.0147 | 8.19% | 0.0053 | 4.27% | 0.0042 | 4.30% |
| Scale5 | 0.0023 | 1.05% | 0.0135 | 6.91% | 0.0031 | 1.44% | 0.0027 | 1.85% |
| Total | 0.0226 | | 0.0514 | | 0.0258 | | 0.0201 | |
| Total Raw | 0.0230 | | 0.0525 | | 0.0262 | | 0.0205 | |
| | FTSE | | IBEX | | MIB | | PSI | |
| Scale1 | 0.0126 | 54.74% | 0.0190 | 46.65% | 0.0218 | 49.69% | 0.0130 | 41.25% |
| Scale2 | 0.0087 | 26.24% | 0.0151 | 29.26% | 0.0165 | 28.45% | 0.0106 | 27.34% |
| Scale3 | 0.0059 | 12.16% | 0.0113 | 16.56% | 0.0121 | 15.45% | 0.0080 | 15.83% |
| Scale4 | 0.0038 | 5.10% | 0.0060 | 4.68% | 0.0064 | 4.27% | 0.0065 | 10.49% |
| Scale5 | 0.0023 | 1.76% | 0.0047 | 2.85% | 0.0045 | 2.15% | 0.0046 | 5.08% |
| Total | 0.0170 | | 0.0278 | | 0.0309 | | 0.0202 | |
| Total Raw | 0.0172 | | 0.0285 | | 0.0314 | | 0.0208 | |

Table 10. Forecast of Beta and R^2 for the second post crisis period.

| | Beta at each scale | | | | | R^2 at each scale | | | | | Raw Data | |
|----------|--------------------|-------|-------|-------|------|---------------------|-------|-------|-------|-------|----------|-------|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R^2 |
| AEX | | | | | | | | | | | | |
| Mean | 1.23 | 1.19 | 1.18 | 1.17 | 1.23 | 0.43 | 0.45 | 0.47 | 0.37 | 0.33 | 1.20 | 0.43 |
| SD | 0.57 | 0.56 | 0.52 | 0.63 | 0.70 | 0.17 | 0.18 | 0.17 | 0.20 | 0.24 | 0.53 | 0.16 |
| Skew. | 0.18 | 0.39 | 0.57 | 1.01 | 0.08 | -0.26 | -0.33 | -0.25 | 0.22 | 0.31 | 0.30 | -0.39 |
| Kurtosis | 1.84 | 2.03 | 2.19 | 3.90 | 1.73 | 2.73 | 2.32 | 2.59 | 2.21 | 1.84 | 2.02 | 2.66 |
| ATHEX | | | | | | | | | | | | |
| Mean | 1.00 | 1.02 | 1.06 | 1.16 | 1.09 | 0.44 | 0.45 | 0.42 | 0.47 | 0.69 | 1.03 | 0.46 |
| SD | 0.57 | 0.61 | 0.62 | 0.78 | 0.45 | 0.19 | 0.18 | 0.21 | 0.26 | 0.17 | 0.57 | 0.17 |
| Skew. | 0.88 | 0.85 | 0.46 | 0.72 | 0.07 | 0.30 | 0.43 | -0.20 | -0.32 | -1.31 | 0.80 | 0.32 |
| Kurtosis | 2.45 | 2.49 | 2.15 | 2.64 | 2.74 | 2.13 | 2.18 | 2.18 | 1.92 | 4.97 | 2.44 | 2.11 |
| CAC 40 | | | | | | | | | | | | |
| Mean | 1.17 | 1.15 | 1.16 | 1.11 | 1.15 | 0.54 | 0.57 | 0.57 | 0.46 | 0.43 | 1.15 | 0.54 |
| SD | 0.39 | 0.39 | 0.44 | 0.55 | 0.55 | 0.15 | 0.15 | 0.16 | 0.19 | 0.21 | 0.39 | 0.15 |
| Skew. | 0.04 | 0.18 | 0.46 | 1.17 | 0.11 | -0.32 | -0.80 | -0.68 | -0.16 | -0.18 | 0.20 | -0.45 |
| Kurtosis | 2.06 | 2.11 | 2.61 | 4.80 | 2.01 | 2.04 | 3.26 | 2.99 | 1.78 | 2.19 | 2.31 | 2.14 |
| DAX 30 | | | | | | | | | | | | |
| Mean | 0.94 | 0.99 | 0.95 | 0.95 | 1.03 | 0.43 | 0.54 | 0.55 | 0.42 | 0.46 | 0.97 | 0.49 |
| SD | 0.33 | 0.34 | 0.32 | 0.35 | 0.39 | 0.16 | 0.15 | 0.17 | 0.18 | 0.17 | 0.31 | 0.15 |
| Skew. | 0.37 | 0.36 | -0.19 | 0.40 | 0.66 | -0.08 | -0.23 | -0.46 | 0.25 | -0.56 | 0.08 | -0.21 |
| Kurtosis | 2.66 | 2.40 | 2.32 | 2.29 | 3.05 | 2.16 | 2.44 | 2.34 | 1.84 | 2.13 | 2.43 | 2.26 |
| FTSE 100 | | | | | | | | | | | | |
| Mean | 1.02 | 1.09 | 1.09 | 1.02 | 0.95 | 0.39 | 0.41 | 0.39 | 0.34 | 0.35 | 1.04 | 0.39 |
| SD | 0.45 | 0.50 | 0.53 | 0.50 | 0.55 | 0.15 | 0.16 | 0.18 | 0.16 | 0.20 | 0.45 | 0.15 |
| Skew. | 0.75 | 0.53 | 0.43 | 0.78 | 0.83 | -0.09 | -0.04 | -0.20 | 0.22 | 0.21 | 0.60 | -0.01 |
| Kurtosis | 3.14 | 2.42 | 2.44 | 2.86 | 4.05 | 2.14 | 2.15 | 2.10 | 2.20 | 1.96 | 2.57 | 2.13 |
| IBEX 35 | | | | | | | | | | | | |
| Mean | 1.00 | 0.96 | 1.03 | 0.89 | 0.96 | 0.54 | 0.59 | 0.60 | 0.43 | 0.56 | 0.99 | 0.56 |
| SD | 0.26 | 0.27 | 0.32 | 0.41 | 0.43 | 0.16 | 0.17 | 0.17 | 0.26 | 0.24 | 0.27 | 0.16 |
| Skew. | 0.16 | -0.03 | 0.85 | -0.09 | 0.18 | 0.88 | 0.60 | 0.34 | 0.24 | -0.32 | 0.04 | 0.81 |
| Kurtosis | 2.45 | 1.60 | 4.53 | 1.71 | 1.69 | 2.93 | 2.54 | 2.36 | 2.04 | 2.24 | 2.09 | 2.66 |
| MIB | | | | | | | | | | | | |
| Mean | 1.07 | 1.05 | 1.07 | 0.98 | 1.11 | 0.52 | 0.54 | 0.54 | 0.47 | 0.47 | 1.06 | 0.52 |
| SD | 0.42 | 0.45 | 0.51 | 0.44 | 0.57 | 0.16 | 0.15 | 0.18 | 0.20 | 0.25 | 0.45 | 0.15 |
| Skew. | 0.57 | 0.35 | 0.36 | 0.68 | 0.26 | 0.18 | -0.02 | -0.30 | -0.29 | -0.32 | 0.40 | 0.08 |
| Kurtosis | 2.27 | 1.89 | 2.33 | 3.84 | 2.08 | 2.14 | 2.03 | 2.73 | 2.06 | 1.68 | 1.98 | 2.18 |
| PSI-20 | | | | | | | | | | | | |
| Mean | 0.93 | 0.90 | 0.94 | 1.16 | 1.06 | 0.23 | 0.28 | 0.32 | 0.41 | 0.45 | 0.97 | 0.29 |
| SD | 0.49 | 0.48 | 0.50 | 0.76 | 0.70 | 0.14 | 0.15 | 0.15 | 0.19 | 0.28 | 0.51 | 0.14 |
| Skew. | 0.02 | 0.31 | 0.94 | 0.77 | 0.13 | -0.03 | -0.07 | -0.53 | -0.72 | -0.04 | 0.40 | -0.15 |
| Kurtosis | 1.93 | 2.17 | 4.05 | 2.98 | 1.68 | 1.78 | 2.07 | 2.45 | 3.00 | 1.65 | 2.46 | 2.19 |

Table 11. Forecast of Value At Risk (VaR) at different time scales for equally weighted portfolio. Second post crisis period.

| | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR | VaR | Contribution to VaR |
|-----------|--------|------------------------|--------|------------------------|--------|------------------------|--------|------------------------|
| | AEX | | ATHEX | | CAC | | DAX | |
| Scale1 | 0.0120 | 46.59% | 0.0265 | 39.86% | 0.0138 | 43.90% | 0.0102 | 38.22% |
| Scale2 | 0.0093 | 27.79% | 0.0208 | 24.67% | 0.0114 | 30.12% | 0.0094 | 32.22% |
| Scale3 | 0.0073 | 17.20% | 0.0157 | 14.02% | 0.0087 | 17.47% | 0.0074 | 19.82% |
| Scale4 | 0.0043 | 6.04% | 0.0139 | 11.05% | 0.0050 | 5.68% | 0.0040 | 5.96% |
| Scale5 | 0.0027 | 2.38% | 0.0135 | 10.39% | 0.0035 | 2.83% | 0.0032 | 3.77% |
| Total | 0.0176 | | 0.0419 | | 0.0208 | | 0.0165 | |
| Total Raw | 0.0181 | | 0.0433 | | 0.0213 | | 0.0171 | |
| | FTSE | | IBEX | | MIB | | PSI | |
| Scale1 | 0.0086 | 46.52% | 0.0152 | 39.20% | 0.0164 | 41.15% | 0.0093 | 31.91% |
| Scale2 | 0.0068 | 28.89% | 0.0136 | 31.36% | 0.0141 | 30.19% | 0.0085 | 26.47% |
| Scale3 | 0.0048 | 14.59% | 0.0107 | 19.43% | 0.0111 | 18.71% | 0.0069 | 17.52% |
| Scale4 | 0.0033 | 6.60% | 0.0058 | 5.76% | 0.0063 | 6.06% | 0.0066 | 16.05% |
| Scale5 | 0.0023 | 3.40% | 0.0050 | 4.25% | 0.0051 | 3.89% | 0.0047 | 8.05% |
| Total | 0.0127 | | 0.0243 | | 0.0256 | | 0.0165 | |
| Total Raw | 0.0130 | | 0.0252 | | 0.0263 | | 0.0171 | |

Appendix

Table 12. Beta and R^2 computed from recomposed crystals of each stock in Portugal index PSI 20. Pre crisis period.

| | Beta at each scale | | | | | R^2 at each scale | | | | | Raw Data | |
|----------|--------------------|-------|-------|------|------|---------------------|------|------|-------|------|----------|-------|
| Symbol | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | Beta | R^2 |
| P:ALT | 1.02 | 1.47 | 0.82 | 0.54 | 0.27 | 0.07 | 0.11 | 0.05 | 0.02 | 0.01 | 1.03 | 0.08 |
| P:BPI | 0.72 | 0.73 | 0.99 | 0.80 | 0.81 | 0.09 | 0.07 | 0.17 | 0.24 | 0.18 | 0.85 | 0.13 |
| P:BCP | 1.46 | 1.36 | 1.49 | 1.23 | 1.49 | 0.36 | 0.33 | 0.49 | 0.44 | 0.54 | 1.35 | 0.38 |
| P:BES | 0.42 | 0.39 | 0.61 | 0.63 | 0.26 | 0.12 | 0.08 | 0.24 | 0.33 | 0.03 | 0.51 | 0.18 |
| P:ECP | 1.10 | 0.99 | 1.04 | 1.04 | 1.05 | 0.34 | 0.28 | 0.42 | 0.46 | 0.37 | 1.07 | 0.38 |
| LX:ESP | -0.13 | -0.18 | -0.06 | 0.63 | 0.37 | 0.00 | 0.01 | 0.00 | 0.12 | 0.04 | 0.02 | 0.00 |
| P:JMT | 0.63 | 0.32 | 0.54 | 0.57 | 0.36 | 0.09 | 0.03 | 0.08 | 0.14 | 0.07 | 0.56 | 0.09 |
| P:EGL | 0.61 | 0.48 | 0.67 | 1.44 | 0.99 | 0.07 | 0.03 | 0.06 | 0.32 | 0.17 | 0.76 | 0.10 |
| P:PTI | 0.39 | 0.30 | 0.69 | 0.69 | 1.40 | 0.03 | 0.02 | 0.09 | 0.16 | 0.34 | 0.56 | 0.07 |
| P:PTC | 1.24 | 1.42 | 0.92 | 1.08 | 0.90 | 0.32 | 0.38 | 0.28 | 0.45 | 0.30 | 1.18 | 0.36 |
| P:SEM | 0.45 | 0.30 | 0.61 | 0.74 | 1.64 | 0.04 | 0.02 | 0.09 | 0.14 | 0.38 | 0.59 | 0.08 |
| P:SNCA | 0.99 | 1.26 | 0.77 | 1.79 | 1.96 | 0.10 | 0.11 | 0.05 | 0.30 | 0.32 | 1.15 | 0.12 |
| P:SOI | 0.62 | 0.67 | 1.14 | 0.85 | 0.66 | 0.05 | 0.05 | 0.13 | 0.13 | 0.08 | 0.79 | 0.09 |
| P:SON | 1.05 | 1.01 | 1.24 | 1.59 | 1.46 | 0.16 | 0.16 | 0.21 | 0.44 | 0.42 | 1.15 | 0.22 |
| P:PTM | 0.35 | 0.54 | 0.65 | 0.96 | 0.84 | 0.05 | 0.09 | 0.18 | 0.41 | 0.20 | 0.56 | 0.13 |
| Mean | 0.73 | 0.74 | 0.81 | 0.97 | 0.96 | 0.13 | 0.12 | 0.17 | 0.27 | 0.23 | 0.81 | 0.16 |
| SD | 0.41 | 0.49 | 0.36 | 0.39 | 0.54 | 0.12 | 0.12 | 0.14 | 0.15 | 0.16 | 0.35 | 0.12 |
| Skew. | -0.12 | 0.03 | -0.38 | 0.80 | 0.24 | 1.15 | 1.17 | 1.05 | -0.10 | 0.18 | -0.44 | 0.12 |
| Kurtosis | 2.57 | 2.01 | 3.80 | 2.49 | 1.99 | 2.81 | 3.02 | 3.20 | 1.63 | 1.89 | 2.77 | 0.12 |